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PREFACE

The purpose of the Conference is to summarize the recent advances, to discuss the status, the development trends and also to share views on the current and future challenges in the wide subject area of stochastic mechanics and, more generally, in applied stochastics.

The development of stochastic methods in mechanics and of stochastic dynamics in particular, has been achieved over the past four decades owing to the research and collective, persistent efforts of many people. However, with no doubt there can be identified a group of world leaders who made mile stone, or corner stone, contributions to the subject area, having laid out the avenues for pursuing research and having inspired the research of many others. One of those leaders is Professor Kazimierz Sobczyk, who and whose research work are worldwide known. It happens that 2009 is the year of his seventieth birthday. This has presented a very special occasion for convening a conference aimed at taking a broader view on the status and future challenges of the subject area of stochastic mechanics.

The present book contains the extended abstracts of 26 papers to be presented at the conference. The papers cover the range of up-to-date topics in applied stochastics and stochastic mechanics as well as some topics in mechanics of materials. These topics are of relevance to Professor Sobczyk areas of research and his pioneering contributions.

This volume of abstracts is complemented with an essay by Professor Kazimierz Sobczyk presenting his way through stochastic mechanics. We also present a list of publications by Professor Kazimierz Sobczyk.

The editors would like to thank all the authors and the attendees of the Conference for their contributions to this event.

The Editors,

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Table of contents

PATTERN RECOGNITION AND STATISTICAL LEARNING IN STOCHASTIC MECHANICS

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1. Introduction

Modern engineering materials often display substantial heterogeneity in their properties at scales that are similar to those at which material damage initiates. Some examples are the variation of crystallographic orientation in polycrystalline metals and the elastic property mismatch between reinforcing particles/fibers and the matrix of composite materials. The efficient and safe use of these materials in demanding engineering applications such as aviation, space, and defense structures requires detailed understanding of the effect of these material heterogeneities on stress and strain fields developed under loading and the resulting effect on material service lifetimes.

Engineering analysis of materials with strong spatial heterogeneity of material properties poses many significant challenges, among which are the randomness of the geometry, the very strong gradients present in the material property fields, and the large difference in scales between the smallest material constituents and the material volumes used in engineering applications. All of these challenges make the use of standard finite element analysis difficult and computationally expensive, often requiring very fine meshes, and, in the case of materials with random microstructures, expensive Monte Carlo simulation to provide statistics of the expected response.

This presentation describes an alternative, approximate, method for the analysis of materials with spatially heterogeneous material properties that makes use of well-developed tools of pattern recognition and statistical learning. The objective is to develop a method that can predict, without solution of the governing equations of elasticity, the location of large elastic stress or strain concentration in a heterogeneous material subject to deterministic boundary conditions.

2. Problem Statement

Let $D \subset R^n$ be a domain occupied by a material with spatially varying elastic properties $C(x)$, subject to Dirichlet or Neumann boundary conditions, or a combination of the two types. The boundary conditions generate a response $r(x)$ in the material which can consists of the stress and strain fields $\sigma(x)$ and $\varepsilon(x)$. The critical regions of the material are those in which a condition of the type

$$r(x) > r_{threshold}$$

is satisfied. That is, locations at which the material response exceeds a threshold value. Examples include conditions on the allowable maximum principal stress or strain, the maximum shearing stress, or any other combination of stress or strain values. Typically, such a condition would be associated with a criterion for the onset of damage in the material. The goal is to identify the critical regions that are defined by

$x \in D_{critical}$ if $r(x) > r_{threshold}$.

3. Methods

The problem stated above is solved in an approximate fashion by detecting patterns in the material property fields that are associated with the criticality condition being met. The first step in the analysis is to identify such patterns through the analysis of a set of training data. These training data typically comprise a set of randomly generated microstructures for which the response field has been calculated using finite element analysis so that for each element of the training set the criticality can be determined.

The second step in the analysis consists of identifying patterns in the set of training samples in which the response is critical. This step can be accomplished by a variety of data mining techniques and the two used here are Principal Components Analysis and analysis by the Sobol' Decomposition. At the end of this stage of analysis a set of basis vectors are established that can be used to represent the spatially varying material properties of the material.

Finally, using the training samples and the new basis vectors, classifiers are developed that predict, based on a projection of a random microstructure onto the new basis vectors, whether the particular microstructural configuration is likely to lead to critical material response that may in turn lead to damage initiation. The classifiers, either support vector machines or decision trees [1], can be implemented in a moving window algorithm to extract $D_{critical}$ from D. The success of the approach can be evaluated by assessing the number of true positive, true negative, false positive, and false negative results. In an engineering context, false negative results, which falsely indicate safety, are non-conservative, and the classifiers can be trained to avoid such errors.

4. Example Applications

This presentation describes the application of the above methods to two example application problems. In both cases the criticality criterion is based on maximum principal stress/strain and the material is assumed to remain elastic when subject to uniaxial extension. The first example considers a two dimension fiber-reinforced composite material [2,3] and the second example considers a two dimension polycrystalline material in which the grains have varying crystallographic orientation [4]. Results in both cases are good, with high true positive and low false negative rates. Figure 1 shows an example result for the polycrystalline case, in which the classifier broadly predicts the locations at which the stress in the material is highly elevated. The black pixels indicate critical locations, and panel (b) shows the prediction while panel (c) shows the 'exact' result obtained by finite element analysis.

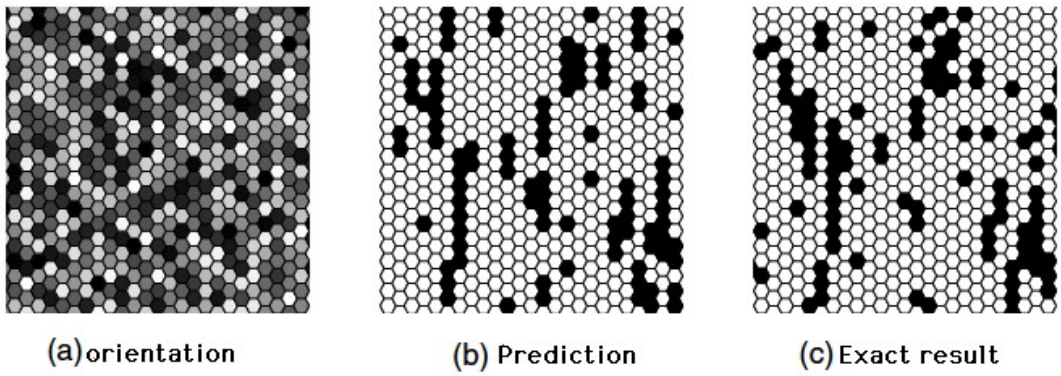

Figure 1: Example classification of polycrystalline microstructure. Orientation variation produces elastic property variation (a), classifier predicts location of stress concentration (b), finite element analysis provides validation result for classifier prediction (c).

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GENERALIZED FPK EQUATIONS FOR NON-LINEAR DYNAMICAL SYSTEMS UNDER GENERAL STOCHASTIC EXCITATION

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We address the problem of determining the probabilistic structure of the response of a dynamical system governed by a differential system of quite general form, with arbitrary polynomial non-linearities, subject to a general stochastic excitation. The latter is assumed to be distributed in accordance with a known probability measure, defined over the Borel $\sigma-$algebra of continuous functions. The goal of this paper is to derive appropriate equations for determining the probability density function of the response, in terms of the probabilistic characteristics of the stochastic excitation. Since the excitation is not a delta correlated process, the response does not obey the Markov property.

Consider, for simplicity, the two-equation differential system

$$\begin{aligned} &\dot{x}_n(t) = \sum_{q_1,q_2} A^{(n)}_{q_1 q_2}(t)\cdot x_1^{q_1}(t)x_2^{q_2}(t) + \sum_{Q_1,Q_2} B^{(n)}_{Q_1 Q_2}(t)\cdot y_1^{Q_1}(t;\theta)y_2^{Q_2}(t;\theta), \\ &x_n(t_0) = x_{n0}(\theta), \qquad n=1,2, \end{aligned} \tag{1}$$

where $A^{(n)}_{q_1 q_2}(t)$ and $B^{(n)}_{Q_1 Q_2}(t)$ are known deterministic functions, $y_1(t;\theta)$, $y_2(t;\theta)$ are given stochastic functions, both defined on the common domain $\mathcal{T}\times\Theta = [t_0, T]\times\Theta$, Θ being the sample space, $x_0(\theta)$ is a given stochastic variable, and q_1, q_2, Q_1, Q_2 are non-negative integers, each one non-exceeding a bounded maximum value. Clearly, both linear and quadratic stochastic excitation are included in the excitation term of the above equation. A special case of particular interest is a linear system driven by quadratic (colored) noise, already examined by Luczka (1986), using methods different from the one presented herewith. See also Luczka, Hänggi & Gadomski (1995).

A general technique to deal with problem (1) is to derive and study the infinite system of moment equations. After truncation this system can become closed (by means of appropriate closure schemes) and solved, providing us with useful (yet restricted) information about the probabilistic characteristics of the response process.

Another general approach to treat problem (1), initiated by Hopf (1952) in the context of his *statistical approach to turbulence*, is to consider the characteristic functional of the response process and find equations governing its evolution (see also Monin & Yaglom 1971, 1975). Due to the complexity of these Functional Differential Equations (FDEs), solutions are known only to very specific cases. An alternative approach, developed by Kotulski & Sobczyk (1984), is to directly construct the characteristic functional of the response process, exploiting the differential equations for sample functions. The latter method, as well as some similar works by Budini & Caseres (e.g., 2004), seem to be generally applicable only to linear problems.

An extension to Hopf's method was presented by Lewis & Kraichnan (1962), who introduced the joint, response-excitation, characteristic functional, and found the corresponding FDEs (see, also, Beran 1968). In the present study we follow the latter

idea, appropriately adapted to problem (1). After deriving the FDEs, we combine them and project appropriately to finite dimensions. The projection is implemented by substituting the general arguments in the characteristic functional by delta functionals, when appropriate. After lengthy calculations we are able to get an equation for the joint, response-excitation (4-dimensional) characteristic function $\varphi_{x_1(t)x_2(t)y_1(s)y_2(s)}\left(u_1,u_2,v_1,v_2\right)$. Applying Fourier transform, we finally derive an equation governing the evolution of the corresponding joint, response-excitation (4-dimensional) probability density function $f_{x_1(t)x_2(t)y_1(s)y_2(s)}\left(\alpha_1,\alpha_2,\beta_1,\beta_2\right)$:

$$\frac{\partial}{\partial t} f_{x_1(t)x_2(t)y_1(s)y_2(s)}\left(\alpha_1,\alpha_2,\beta_1,\beta_2\right)\Big|_{s\to t} + \mathscr{L}_{\alpha_1\alpha_2}\left[f_{x_1(t)x_2(t)y_1(t)y_2(t)}\left(\alpha_1,\alpha_2,\beta_1,\beta_2\right)\right] + \\ + \sum_{n=1,2}\sum_{Q_1,Q_2} B^{(n)}_{Q_1Q_2}\beta_1^{Q_1}\beta_2^{Q_2}\frac{\partial}{\partial a_n} f_{x_1(t)x_2(t)y_1(t)y_2(t)}\left(\alpha_1,\alpha_2,\beta_1,\beta_2\right) = 0 \tag{2}$$

where $\mathscr{L}_{\alpha_1\alpha_2}[\bullet]$ is a first-order differential operator with respect to α_1,α_2, defined by

$$\mathscr{L}_{\alpha_1\alpha_2}[\bullet] = \sum_{n=1,2}\sum_{q_1,q_2} A^{(n)}_{q_1q_2}\alpha_1^{q_1}\alpha_2^{q_2}\left(\alpha_n^{-1}q_n + \frac{\partial}{\partial\alpha_n}\right)[\bullet]. \tag{3}$$

[For $q_n = 1$ the term α_n^{-1} should be replaced by 1]. Equation (2) should be supplemented by appropriate initial and marginal-compatibility conditions. It generalizes a similar result for a single scalar equation, presented in Sapsis & Athanassoulis (2008).

The validity of this new equation is assured by showing that the infinite system of the moment equations can be directly derived from it. Further, it should be noted that equation (2) applies to any kind of stochastic excitation, with continuous sample functions. No specific simplifying assumptions, concerning either the correlation structure or the distributions of the stochastic data, are needed. Because of the generic nature of the excitation, the response is non-Markovian. Thus, equation (2) can be considered as a generalization of the FPK equation to a broad class of stochastic dynamical systems, exhibiting non-Markovian responses. Generalizations of the FPK equation for specific systems exhibiting non-Markovian responses have also been presented by many authors. See, e.g., Luczka, Hänggi & Gadomski (1995) and the survey by Luczka (2005).

Equation (2) does not belong to any, already studied, type of partial differential equations. Its solvability theory and appropriate methods for its effective numerical solution should by developed. A particular method for its numerical solution is under development and will be presented.

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Solution of the first passage problem by asymptotic sampling

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1. Introduction

The first passage problem in random vibrations is readily written as a high-dimensional reliability problem. The the first passage probability or probability of failure P_F in an n-dimensional space of random variables $X_1, \ldots X_n$ can be computed as

$$P_F = \int_{D_F} \cdots \int f_{X_1,\ldots X_n}(x_1, \ldots x_n) \mathrm{d}x_1 \ldots \mathrm{d}x_n$$

In this equation, $f_{X_1,\ldots X_n}(x_1, \ldots x_n)$ denotes the joint probability function of the random variables $X_1, \ldots X_n$ and D_F denotes the failure domain, i.e. the region of the n-dimensional random variable space in which failure occurs. In the context of the first passage problem, this denotes combinations of input variables such that a response variable exceeds a critical threshold value. The generalized safety index (or reliability index) β is defined by

$$\beta = \Phi^{-1}(1 - P_F)$$

Here $\Phi^{-1}(.)$ is the inverse standardized Gaussian distribution function. In [2,3], a novel method called *Asymptotic sampling* is presented which avoids some of the drawbacks associated with high-dimensional reliability analysis. The underlying concept relies on the asymptotic behavior of the failure probability in n-dimensional i.i.d Gaussian space as the standard deviation σ of the variables and hence the failure probability P_F approaches zero (see e.g. [1]). Consider a (possibly highly nonlinear) limit state function $g(\mathbf{X})$ in which $g < 0$ denotes failure. Let σ be the standard deviation of the i.i.d. Gaussian variables $X_k, k = 1 \ldots n$. It is attempted to determine the functional dependence of the generalized safety index β on the standard deviation σ or its inverse $f = \frac{1}{\sigma}$ by using an appropriate sampling technique. One major advantage of this approach is its independence of the dimensionality n.

2 Numerical example

This example a single-degree-of-freedom structural model with a non-linear hysteretic restoring force according to the well-known Bouc-Wen model. This structure is subject to an earthquake-type ground excitation. The excitation model used in this example is simply an amplitude-modulated white noise (shot noise). Based on this model, the earthquake excitation $a(t)$ is generated as

$$a(t) = e(t)w(t)$$

in which $w(t)$ is white noise with intensity D_0, i.e. $R_{ww}(\tau) = D_0\delta(\tau)$, $e(t)$ is a modulating function, here chosen as

$$e(t) = 4 \cdot [\exp(-0.25t) - \exp(-0.5t)]$$

In order to apply this approach in digital simulation, the continuous time white noise excitation needs to be discretized. This is achieved by representing the white noise $w(t)$ by a sequence of i.i.d. random variables $W_k, k = 1 \ldots m$ assumed to be constant values spaced at time intervals Δt. The number of random variables representing the white noise is chosen as N = 1000. The total time duration is T = 20 s, so that the time interval is $\Delta t = \frac{T}{N}$ = 0.02 s. The structural model is assumed to have one kinematic degree of freedom $x(t)$. In addition, there is an internal plastic displacement variable $z(t)$ describing the plastic behavior of the structure. The structural model has a mass m. The equation for the derivative $\dot{z}$ of the plastic variable depends on the state of the system. For the Bouc-Wen model this is defined by the differential equation

$$\dot{z} = A\dot{x} - \beta\dot{x}|z| - \gamma|\dot{x}|z$$

For the state variable x we have the equations of motion:

$$m\ddot{x} + c\dot{x} + (1-\alpha)kz + \alpha kx = -ma(t)$$

Here c is a viscous damping factor. The numerical values used in this example are k = 1 MN/m, m = 40 t, c = 5 kNs/m, α = 0.603, β = -1.8548, γ = 39.36, A = 5.868. The equations of motion are rewritten in first-order form and then numerically. Carrying out the asymptotic sampling procedure for a displacement threshold of Ω = 0.5 m yields the first passage probabilities as shown in Table 1. For reference, Monte Carlo simulation with one million samples yields the result β_{MC} = 3.75.

Table 1: Asymptotic sampling results for different number M of sample points

M	100	200	500	1000
β	3.35	3.76	3.80	3.70

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STOCHASTIC ROTORDYNAMICS: DIRECT AND INVERSE PROBLEMS

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Predicting transverse random vibrations of shafts in rotating machinery may be of importance for applications with high environmental dynamic loads on supports, e.g. in transport. Design of a turbopump for liquid-propellant rocket engine may be quoted as an example whereby operation of the shaft close to its instability threshold was of concern because of increased sensitivity of the whole system shaft-machine-vehicle to such loads [1]. On the other hand, small random vibration components may sometimes be observed in stationary fluid rotating machinery (turbines, fans, etc) – see Figure 1 [2]. The measurable random vibration signals (e.g. due to turbulence in working fluid) may then be used with advantage for on-line condition monitoring of the shaft during its steady operation at a given rotation speed. Survey of recent research results [3 – 8] in transverse random vibrations of rotating shafts is presented here with solutions to various direct and inverse random vibration problems for simple single-disk shafts with potential instability due to internal or "rotating" damping (the latter may also provide a simplified representation for destabilizing nonconservative fluid or magnetic forces).

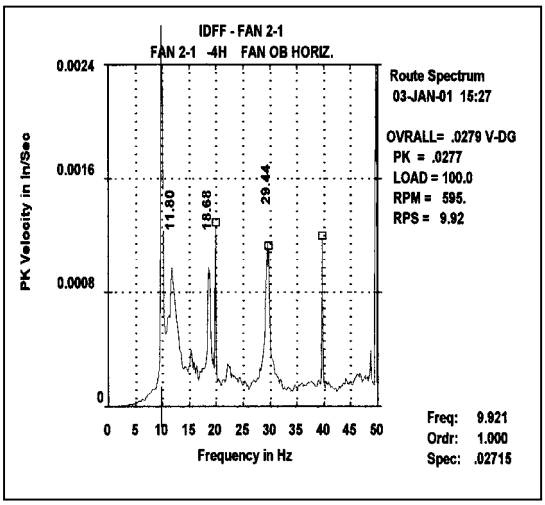

Fig. 1. Spectral density of vibration signal from bearing of a large fan with dominant peak at rotational frequency 9.92 Hz and neighboring peak at the shaft's resonance (critical speed) at 11.80 Hz [2].

Transverse random vibrations of a single-disk two-degrees-of-freeedom rotating shaft with both external ("nonrotating") and internal ("rotating") damping are considered with the latter type of damping being a potential source for dynamic instability. Analytical solutions for mean square transverse displacements $\left\langle X^2 \right\rangle, \left\langle Y^2 \right\rangle$ are obtained for linear vibrations during operation below instability threshold as well as for tilting oscillations with gyroscopic effect of the disk being represented [3]. The results illustrate: i) magnification of the response intensity with increasing rotation speed ν ; thus "universal" magnification law for the mean square whirl radius $\left\langle R^2 \right\rangle = \left\langle X^2 \right\rangle + \left\langle Y^2 \right\rangle$ is

found as $\left\langle R^2 \right\rangle = \left\langle R^2 \right\rangle_{\nu=0} \cdot \left(1-\nu/\nu_*\right)^{-1}$ where ν_* is the instability threshold rotation speed; ii) general trend towards equalizing partition of response energy between two perpendicular directions: even in case of uniaxial excitation ratio of mean square responses in nonexcited and excited directions approaches unity with $\nu/\nu_* \to 1$ that is, with approaching state of forward whirl. Furthermore, coherence functions of responses in two perpendicular directions are calculated as functions of rotation speed [3, 8]. These results are used to develop procedures for on-line evaluation of the shaft's stability margin that rely on response signals measurements during steady operation at any rotation speed below ν_*.

The shaft is also considered for the case where its "rotating" damping is subject to slow temporal random variations that may lead to potential "short-term" instability [6]. The corresponding transient response analysis as based on the Krylov-Bogoliubov averaging and parabolic approximation for peaks of the random damping factor provides probabilistic predictions for outbreaks in the shaft's radius of whirl. Procedure for estimating statistical properties of damping variations from the observed intermittent response with outbreaks, or "puffs" is outlined also.

Lateral vibrations of a single-disk shaft are considered with stiffening nonlinearity taken into account either in restoring force or in damping [4, 5, 7]. Certain exact and approximate analytical solutions for joint probability density function (PDF) of displacements and velocities in two perpendicular directions are obtained. The results may be used to evaluate, from on-line response measurements, whether the shafts operates below or above its instability threshold ν_*. Specifically, PDF *w(V)* of the squared whirl radius $V = X^2 + Y^2$ should be measured for randomly vibrating shaft. If *w(V)*is monotonically decreasing then the shaft is stable in the linear approximation, otherwise its observed response represent self-excited oscillations with superimposed random vibrations.

Finally, first-passage problem is considered for a lightly damped nonlinear shaft. The equations of motion are reduced using stochastic averaging. Then analytical solution for the expected time for crossing giving level by the whirl radius is derived. It can be applied for the important case where stable self-oscillations of the shaft may exist within some range of rotation speeds below instability threshold so that random excitation may lead to a "hard" self-excitation of whirl.

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FRACTIONAL CALCULUS AND PATH INTEGRAL METHOD FOR NONLINEAR SYSTEMS UNDER WHITE NOISE PROCESSES

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1. Introduction

One of the most challenging problem in the field of nonlinear systems under stochastic agencies is to find statistics of the response process. In this framework the Path Integral (PI) method is an effective tool to provide a step-by-step integration technique in terms of Probability density function [see e.g. [1]]. The method starts from the Chapman Kolmogorov equation combined with the so called short-time Gaussian approximation (STGA). Keeping this in mind, the probability density function (PDF) at time $(t+\tau)$ is a convolution integral involving the PDF at time t and a Gaussian kernel that represents the conditional density of the response PDF. The latter is the solution of the Fokker Planck equation with assigned deterministic initial condition at time t, that may be easily evaluated by considering that for (τ) small such a response is Gaussian distributed (STGA). Even though the formulation is quite simple numerical problems arise to evaluate the convolution integrals especially for numerous degree of freedom systems. Moreover working in terms of response moments is impossible since an hierarchy of moments immediately appears.
Recently it has been shown that functional moments that is moments of the type $E[iX^{-\gamma}]$, $\gamma \in C$, $0<\mathrm{Re}(\gamma)<1$ always produce a representation of both PDF and Characteristic Function (CF) in the whole domain of existence of the two domains. It has also been shown that $E[-iX^{-\gamma}]$ coincides with the Riemann Liouville fractional derivative of the CF in zero. By using Melling transform it has also been shown that

$$\phi_X(\pm\vartheta) = \frac{1}{2\pi i}\int_{\rho-i\infty}^{\rho+i\infty}\Gamma(\gamma)E\left[(\mp iX)^{-\gamma}\right]\vartheta^{-\gamma}d\gamma \tag{1}$$

$$p_X(x) = \frac{1}{2\pi^2 \mathrm{i}}\mathrm{Re}\left\{\int_{\rho-\mathrm{i}\infty}^{\rho+\mathrm{i}\infty}\Gamma(\gamma)\Gamma(1-\gamma)E\left[(-\mathrm{i}X)^{-\gamma}\right](\mathrm{i}x)^{\gamma-1}\,\mathrm{d}\gamma\right\} \tag{2}$$

Where $\phi_X(\vartheta)$ is the CF, $p_X(x)$ is the PDF, $\rho = \mathrm{Re}(\gamma)$ and $\Gamma(.)$ is the Euler Gamma function. It has to be remarked that in eq. (1) and (2) integrals are performed along the imaginary axis and then the remain finite also for α - stable processes, for which the integer moments of order greater than two remain divergent ones. Discretization of integrals (1) and (2) produces quite good results as demonstrated in [1]. Keeping these results in mind in the paper it is shown that the PIS remains an amenable problem also from computational point of view [2, 3]. In the latter approaches the PIS was implemented in terms of fractional moments. In the proposed paper a different strategy is proposed to evaluate probability density function and CF at time $(t+\tau)$ by knowing the PDF or the CF at the previous time instant t.

In order to aim at this as a first step the PIS is converted in terms of CF by means of the Fourier transform of the Chapman Kolmogorov equation, then the CF at time t is converted in terms of fractional moments as shown in eq.(1) and then the fractional moments at time $(t+\tau)$ may be easily evaluated by eq.(2).

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STATISTICS OF WAITING TIMES BETWEEN SUDDEN CLIMATE CHANGES AS A TOOL FOR IDENTIFYING POSSIBLE CAUSES

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Sudden climate shifts between two different climatic states have been identified in ice core records covering the last ice age. There has been a long-standing argument among climate scientists on whether these shifts are due to some hitherto unknown external periodic influence maybe from the Sun, or if they are due to internal chaotic fluctuations. In the latter case these fluctuations would most likely occur erratically, with a "memory" reflecting the short time scales of the weather fluctuations.

The erratic appearance of the oxygen isotope record ($\delta^{18}O$) shown and explained in the figure below makes it natural to think of the record as a random response of a stochastic noise driven system. However, the saw tooth character of the sample curve makes it not quite obvious how such a system can be identified from the sample. While the oxygen isotope record is a proxy for the temperature (first pointed out by Dansgaard) there are other climate proxies embedded in the ice cores. One of these is the dust sedimentation recognized by the variation of the calcium ion concentration down through the ice cover as shown in the bottom panel of the figure [log (Ca)]. The DO-events are clearly recognized in this dust record, and no obvious saw tooth behavior is seen.

In fact, a quite good suggestion of a stochastic model is given in [1]. The model gives responses with statistical properties close to those of the dust record. It is shown that the marginal distribution (after suitable first and second moment normalization of the record) as well as the DO-event behavior are well modeled by the stochastic differential equations $\mathrm{d}Y = -(\mathrm{d}U/\mathrm{d}Y)\,\mathrm{d}t + \sigma_1\,\mathrm{d}X + \sigma_2\,\mathrm{d}L$, $\mathrm{d}X = -X\,\mathrm{d}t + \sqrt{1+X^2}\,\mathrm{d}B$, where U is a bistable potential inferred from the data, $\mathrm{d}B$ is standard Brownian noise, $\mathrm{d}L$ is Levy noise with stability index $\alpha = 1.75$, and $\sigma_1/\sigma_2 = 3$. The stationary marginal distribution of the X-process is the Cauchy distribution with density proportional to $1/(1+x^2)$.

It is a challenge to find a similar model for the oxygen isotope record. The problem is to capture the gradual cooling that takes place after each DO-event (the saw tooth behavior). Due to strong negative correlation between the $\delta^{18}O$-record and the log(Ca)-record (even though they are from two different locations) it seems reasonable to look for an extension of the established stochastic differential equation model for the log(Ca)-record.

However, before going on in this direction some preliminary much simpler statistical investigations of the time point series of the DO-events are highly relevant. Even though the analysis is simple and within the toolbox of the elementary statistics textbook, some points of the analysis are interesting. In particular the problem that appear because of the small sample size deserves attention. The simplest possible well fitting model of memoryless random point generation turns out not to be distinguishable from a model built on random deviations from a periodic forcing of the DO-events. It seems that the principle of simplicity of description (known as Occam's razor) is the only way to favor the assumption of generation by pure randomness.

This will be the topic of the presentation even though the work has already been published thanks to a quite fast publication policy of the Journal of Climate [2].

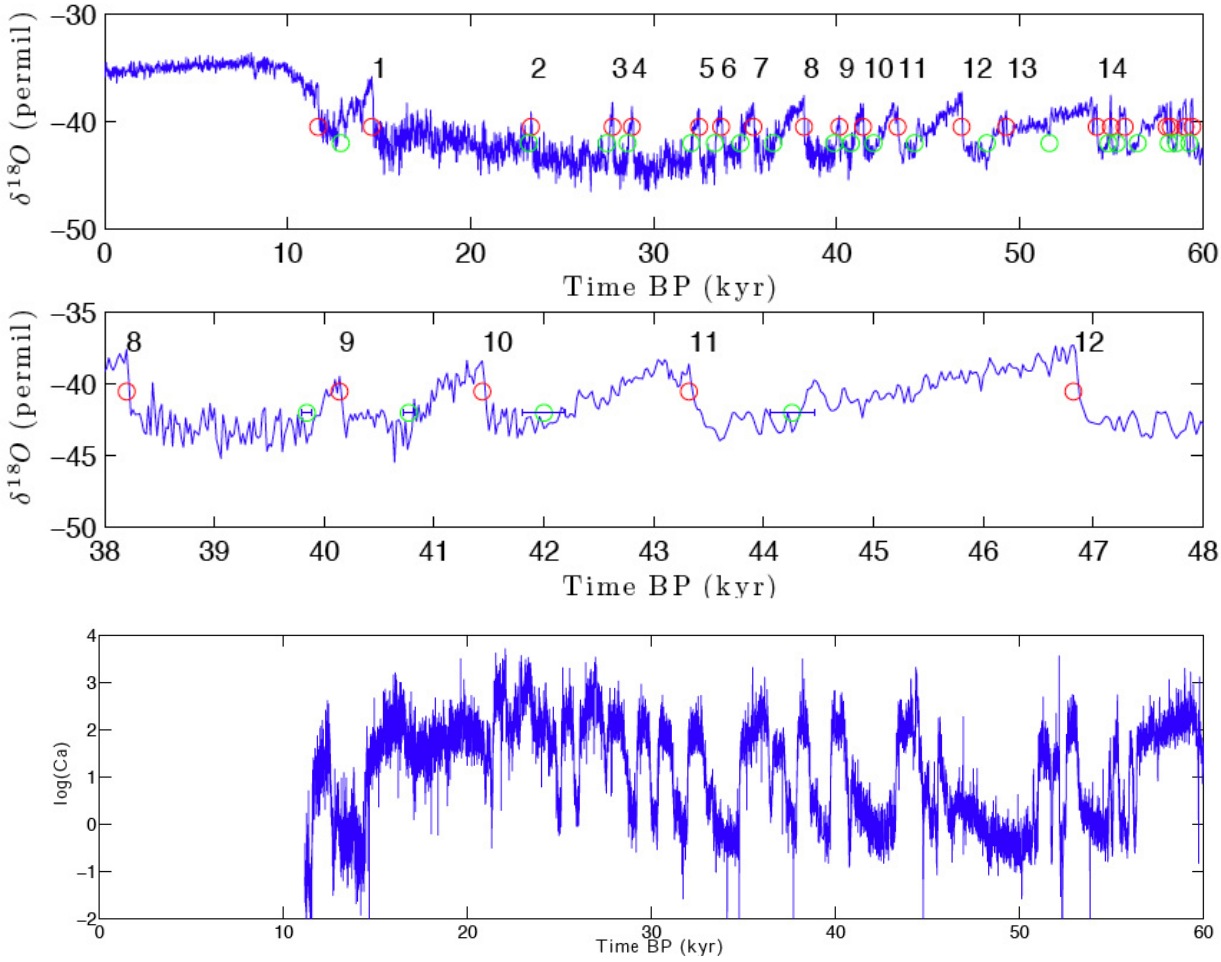

Figure 1: The upper panel shows the $\delta^{18}O$ oxygen isotope record measured down through the ice cover at a given location in central Greenland (the NGRIP ice core project). The upper circles mark the transitions from the stadial cold climate to the interstadial warm climate (off-on points). These are the so-called Dansgaard-Oeschger (DO) events. The lower circles mark the transitions back from the interstadial to the stadial climate (on-off points). The middle panel zooms into the period 38-48 kyr BP, where it is seen that the determination of the on-off transitions are much more uncertain than determining the sharp off-on transitions. This is indicated by errorbars. The bottom panel shows the logarithm of the calcium ion concentration as a function of time in the GRIP ice core. The temporal resolution is about 1 year, much better than for the $\delta^{18}O$ record that suffers from the effect of vapor diffusion. (For clarification it should be noted that the time scales in the top and the bottom panels differ by some factor implying that the DO events are not at the same nominal time on the two scales. Thus the numerically largest correlation is obtained with some linearly increasing time shift).

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STOCHASTIC HOMOGENIZATION FOR CHAOTIC AND QUASI-PERIODIC MASONRY STRUCTURES

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1. Introduction

Ancient masonry structures are often characterized by a chaotic distribution of stones in the walls. In other cases, the shape of the stones and their disposition are such that the structure is called quasi-periodic. In these circumstances, it has been recognised that the best way for defining the mechanic characteristics of the structure is the stochastic one, through the use of the random fields. In fact, the deterministic homogenization, often used for the classical periodic masonry structures, fails for chaotic and quasi-periodic masonry structures and the stochastic homogenization must be considered. In the literature, the stochastic homogenization is referred to the first order statistics, both for the mechanic properties and for the response quantities (displacements and internal forces). It can be applied by using some different approaches: the stochastic convergence approach [1], the polarization tensor approach coupled with the Hashin-Shtrikman variational principle [2] and the concentration tensor approach coupled with the Eshelby equivalence principle. Some approaches in literature consider higher order statistics, but they cannot be considered as homogenization approaches. They are referred as Stochastic Finite Element (SFE) approaches; the most used are: the stochastic perturbation methods [3] and the series expansion approaches, among which the most known is the Karunen-Love series method coupled with the polynomial chaos approach [4].

In the present work a stochastic homogenization approach based on the second order statistics is presented. It is founded on the extension to the second order analyses of the Moving Window Method (MWM), that has been used in the first order stochastic homogenization approach [1,5]. In particular, the extensions of the Voigt and Reuss limits and of the Hill theorem will be considered.

2. Basic formulation of the first order stochastic homogenization

Under the assumptions of stochastic homogeneous and ergodic medium, the average elastic constitutive equation can be written as:

$$\mathsf{E}\left[\sigma_{ij}\left(\mathbf{x}\right)\right]=\mathsf{E}\left[C_{ijkl}\left(\mathbf{x}\right)\varepsilon_{ij}\left(\mathbf{x}\right)\right] \tag{1}$$

where $\mathsf{E}\left[\left(\square\right)\right]$ indicates the mean of $\left(\square\right)$, while σ_{ij}, ε_{ij} and C_{ijkl} are the stress, strain and stiffness tensors. The classical homogenization approach searches for that ideal materials for which the following relationship holds:

$$\mathsf{E}\left[\sigma_{ij}\left(\mathbf{x}\right)\right]=C^{(h)}_{ijkl}\mathsf{E}\left[\varepsilon_{ij}\left(\mathbf{x}\right)\right] \tag{2}$$

If the strains are assumed constant, $\varepsilon_{ij}\left(\mathbf{x}\right)=\varepsilon^{(o)}_{ij}$, then eq.(1) gives

$$\mathsf{E}\left[\sigma_{ij}\left(\mathbf{x}\right)\right]=\mathsf{E}\left[C_{ijkl}\left(\mathbf{x}\right)\right]\varepsilon^{(o)}_{ij} \quad \Rightarrow \quad C^{(h)}_{ijkl}\equiv\mathsf{E}\left[C_{ijkl}\left(\mathbf{x}\right)\right]\equiv C^{(V)}_{ijkl} \tag{3}$$

$C_{ijkl}^{(V)}$ being the Voigt interpretation of the homogenized stiffness tensor. On the contrary, if the stresses are assumed constant, $\sigma_{ij}(\mathbf{x}) = \sigma_{ij}^{(o)}$, then

$$\mathsf{E}\left[\varepsilon_{ij}(\mathbf{x})\right] = \mathsf{E}\left[D_{ijkl}(\mathbf{x})\right]\sigma_{ij}^{(o)} \quad \Rightarrow \quad C_{ijkl}^{(h)} \equiv \left(\mathsf{E}\left[D_{ijkl}(\mathbf{x})\right]\right)^{-1} \equiv C_{ijkl}^{(R)} \tag{4}$$

D_{ijkl} being the point compliance tensor and $C_{ijkl}^{(R)}$ the Reuss interpretation of the homogenized stiffness tensor. The Hill theorem ensures that $\left|C_{ijkl}^{(R)}\right| \le \left|C_{ijkl}^{(h)}\right| \le \left|C_{ijkl}^{(V)}\right|$, the equality sign being strictly verified only when the reference volume of the structure is infinite.

For two finite reference volume $\Omega_1 < \Omega_2$, indicated with $C_{ijkl}^{(a,\Omega_i)}$ and $C_{ijkl}^{(b,\Omega_i)}$ the average stiffness tensors obtained with constant strains and stresses, respectively, then it has been shown that the following fundamental relationship holds [6]:

$$\left|C_{ijkl}^{(b,\Omega_1)}\right| \le \left|C_{ijkl}^{(b,\Omega_2)}\right| \le \left|C_{ijkl}^{(h)}\right| \le \left|C_{ijkl}^{(a,\Omega_2)}\right| \le \left|C_{ijkl}^{(a,\Omega_1)}\right| \tag{5}$$

This last relationship is fundamental in the estimation of the homogenized stiffness tensor in terms of mean values. An effective approach for the evaluation of $C_{ijkl}^{(a,\Omega_i)}$ and $C_{ijkl}^{(b,\Omega_i)}$ is the MWM.

3. Proposed approach

The aim of the present work is the extension of the result summarized in the previous section to the second order statistics. This means that one will work in terms of the correlation function

$$\mathsf{R}_{\substack{ijkl \\ mnpq}}^{(C)}(\mathbf{x}_2 - \mathbf{x}_1) = \mathsf{E}\left[C_{ijkl}(\mathbf{x}_1)C_{mnpq}(\mathbf{x}_2)\right] - \mathsf{E}\left[C_{ijkl}(\mathbf{x}_1)\right]\mathsf{E}\left[C_{mnpq}(\mathbf{x}_2)\right] \tag{6}$$

For example, in these terms, the homogenized stiffness tensor will be that tensor $C_{ijkl}^{(h2)}$ satisfying the following relationship:

$$\mathsf{R}_{\substack{ij \\ mn}}^{(\sigma)}(\mathbf{x}_2 - \mathbf{x}_1) = \mathsf{R}_{\substack{ijkl \\ mnpq}}^{(C^{(h2)})}(\mathbf{x}_2 - \mathbf{x}_1)\mathsf{R}_{\substack{kl \\ pq}}^{(\varepsilon)}(\mathbf{x}_2 - \mathbf{x}_1) \tag{7}$$

that is an extension of eq.(2). In a similar way, it will be shown as the results given into eqs(3-5) are extended to the second order statistics.

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RANDOM HYDROGEN-ASSISTED FATIGUE CRACK GROWTH IN STEEL PLATES

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1. Introduction

Hydrogen lowers the endurance of steel. In particular, it accelerates fatigue crack growth, thereby shortening fatigue lifetime of structures [1]. In the present paper a simple model of the influence of hydrogen on fatigue crack growth is presented. The model is then employed to perform a stochastic analysis of crack growth in a thin plate with a random hydrogen content.

2. Description of the problem

The analysis is performed in the simple case of an infinite, thin plate with a rectilinear crack of length $2a$. The plate contains hydrogen and is subjected to a remote, cyclic loading $S(t)$, perpendicular to the crack. It is assumed that diffusion is very slow, so that hydrogen concentration in the plate does not change significantly during crack propagation. There are small variations in the hydrogen content of the plate, resulting from hydrogen trapping and material inhomogeneities. From the stochastic point of view, hydrogen concentration in the plate is described by a two-dimensional stationary random field $C(x,y,\gamma)$, where x, y are the coordinates on the plate, and γ is the parameter of randomness.

3. Mechanical model

Empirical equations of fatigue crack growth in hydrogen free metals usually take the form [2]

$$\frac{da}{dN} = f(K_{\max}, K_{\min}, material\ \ parameters) \tag{1}$$

where N is the number of load cycles, and $K_{\max}$, $K_{\min}$ are the maximum and minimum stress intensity factors at the crack tip over a cycle of loading. It is postulated that, in the presence of hydrogen, fatigue crack growth can also be described by equations of type (1), after suitably modifying the stress intensity factors. The following assumptions are made:

- There is a Barenblatt-Dugdale cohesive zone in front of a crack tip,
- Hydrogen degradation follows the hydrogen enhanced decohesion mechanism, with a linear dependence of cohesive forces on hydrogen concentration [3],
- Hydrogen concentration is approximately constant in the cohesive zone,
- Residual stresses along the crack are negligible; fatigue crack growth is only influenced by hydrogen degradation of the crack tips [4].

For the considered geometry, the modified mode I stress intensity factors take the form

$$K = \frac{\sigma_0 S}{\sigma_0 - \alpha C}\sqrt{\pi a} \tag{2}$$

where σ_0 is the cohesive force in the absence of hydrogen, α is the coefficient of degradation, and C is the hydrogen concentration in the cohesive zone. It is assumed that fatigue crack growth in the presence of internal hydrogen can be adequately described by combining equations (1) and (2).

4. Stochastic analysis

The presented model allows a convenient stochastic analysis of crack growth in the random hydrogen field $C(x,\gamma)$ - here restricted to one dimension, along the extension of the crack. $C(x,\gamma)$ has a constant mean $\overline{C}$ and a correlation function $K_C(x_1,x_2)$.

A simple example problem can be obtained by specifying (1) to the Paris equation

(3) $$\frac{da}{dN} = A\Delta K^m$$

where $\Delta K = K_{\max} - K_{\min}$, and A, m are constants of the hydrogen-free material. Equation (2) gives

(4) $$\Delta K = \frac{\sigma_0 \Delta S}{\sigma_0 - \alpha C}\sqrt{\pi a} = \frac{\Delta S}{1-\xi}\sqrt{\pi a}$$

where $\Delta S = S_{\max} - S_{\min}$ is the amplitude of the cyclic loading S, and $\xi = \alpha C / \sigma_0$ is a stationary random field. The following treatment is similar to the one presented in [5].

From (3) and (4) one receives (after noting that - with high probability - $\xi << 1$, and by making a linear approximation)

(5) $$dN = B(1-\xi)^m a^{-m/2} da \approx [x + y(\xi - \bar{\xi})] a^{-m/2} da$$

where $B = 1/(A\Delta S^m \pi^{m/2})$, $x = B(1-\bar{\xi})^m$, $y = -Bm(1-\bar{\xi})^{m-1}$, and $\bar{\xi} = \alpha \overline{C} / \sigma_0 = E(\xi)$.

From equation (5) it is easy to obtain the basic probabilistic characteristics of the critical number of cycles N_{cr} (number of cycles to failure). The mean and variance are

(6) $$E[N_{cr}] = E[\int_{a_0}^{a_{cr}} dN] = n\,B(1-\bar{\xi})^m [a_{cr}^{1/n} - a_0^{1/n}]$$

(7) $$Var[N_{cr}] = E[(N_{cr} - E[N_{cr}])^2] = z^2 \int_{a_0}^{a_{cr}}\int_{a_0}^{a_{cr}} (a_1 a_2)^{-m/2} K_C(a_1, a_2)\, da_1 da_2$$

where $m \neq 2$, $n = 2/(2-m)$, $z = y\gamma/\sigma_0$, and a_0, a_{cr} are the initial and critical crack lengths, respectively. Generally, equation (7) must be computed numerically.

In the case when $C(x,\gamma)$ is a Gaussian process, N_{cr} is by equation (5) also Gaussian, and therefore is described completely by the mean and variance given in (6) and (7). The accuracy of this result is limited by the linear approximation made in equation (5) and by the fact that $C(x,\gamma)$ must be positive and so can not be strictly Gaussian.

5. Conclusions

A method of describing fatigue crack growth in thin steel plates containing hydrogen was briefly outlined. The method is convenient in performing stochastic analysis of crack propagation, when hydrogen concentration in the plate is described by a random field.

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INTEGRO-DIFFERENTIAL CHAPMAN-KOLMOGOROV EQUATION FOR CONTINUOUS-JUMP MARKOV PROCESSES AND ITS USE IN PROBLEMS OF MULTI-COMPONENT RENEWAL IMPULSE PROCESS EXCITATIONS

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1. Introduction

State vector of a dynamic system under a Poisson train of impulses is a non-diffusive Markov process and its joint probability density function satisfies an integro-differential generalized
Fokker-Planck-Kolmogorov equation which is also called Kolmogorov-Feller equation (cf. e.g. [1]). If the train of impulses is driven by non-Poisson, for example renewal, counting processes, the state vector is not a Markov process. For some classes of non-Poisson counting processes the non-Markov pulse problems can be converted into the Markov ones by augmenting the state vector of the dynamic system by auxiliary variables driven by either a single or two independent Poisson processes. Exact techniques of this kind have been developed for trains of impulses driven by Erlang renewal processes [2,3] or by a generalized Erlang renewal process [4]. As the augmented state vector is Poisson-driven, the differential equations for response moments can be derived. In all those impulse process excitations problems the displacement response is continuous, the velocity response is continuous-jump and the auxiliary variables are purely jump processes. As those auxiliary jump processes are driven by Poisson processes, they can be alternatively characterized in terms of a number of Markov states. Hence, the problem is jointly described by the original state variables and the Markov states of the auxiliary jump process. Accordingly, the response probability distribution may be characterized by a joint probability density-distribution of the response variables and of the states of a pertinent Markov chain. The fundamental equation for continuous-jump Markov processes is the general integro-differential Chapman-Kolmogorov equation [5]. The explicit integro-differential equations governing the joint probability density-distribution of the response are obtained from the Chapman-Kolmogorov equations, after the determination of the jump probability intensity functions for the continuous-jump and purely jump processes. The explicit equations governing the response probability density have been derived for oscillators under random trains of impulses driven by single, renewal processes [6,7]**.**

2. Statement of the problem

In the present paper the approach to the excitation impulse process based on the integro-differential Chapman-Kolmogorov equation is extended from a single renewal impulse process to a a multi-component one. First, the integro-differential Chapman-Kolmogorov equation is introduced and its use for purely jump processes is demonstrated. The examples of purely jump stochastic processes are the Poisson counting process and a two-state Markov process (a rectangular wave process). The considered impulse excitation consists of a number of **n** random trains of impulses, each of whom is driven by an Erlang renewal process with parameter k(i). The driving processes are assumed to be statistically independent. Each of the impulse processes is recast into a Poisson driven impulse process, with the aid of auxiliary, purely jump stochastic variables, each of whom is governed by a stochastic differential equation driven by the i-th Poisson process.

As each i-th Erlang impulse process is characterized by k(i) "phases", a chain of k(i) Markov states is associated with it. The Markov chain for the whole problem is constructed by considering the coincidences of the states of the individual jump processes. Thus the total number of Markov states is determined. The jump probability intensity functions pertinent to this problem are formulated. The explicit equations governing the joint probability density-distribution function of the response and of the Markov states of the auxiliary jump variables are derived from the general integro-differential forward Chapman-Kolmogorov equation via the integrations over the state space. The resulting equations form a set of integro-partial differential equations.

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REPUTATION AS OPTIMALITY MEASURE IN WIRELESS SENSORS NETWORKS (WSN)-BASED MONITORING SYSTEMS

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1. Wireless Sensors Networks

Many solutions that wireless technology offers for practical engineering problems are based on Wireless Sensors Networks (WSN). These are networks consisting of small devices, usually autonomously controlled and with their own energy resources, equipped with different kinds of detectors. These devices with measuring detectors play a role of nodes of the network. Except of a measuring function, nodes also perform data processing and a communication function. WSN are useful for constructing efficient and cost-effective measuring and diagnostic systems. A module-based structure of a WSN makes it possible to connect to as a measuring device many different sensing elements, both analogue and digital. This, in turn, makes WSNs a cheap and flexible measuring tool. Moreover, since WSNs in a communication layer work as P2P or ad-hoc networks, they have adaptive capabilities in diversified working conditions and are capable of transmitting and forwarding data over large distances. Weak points of WSNs' nodes are their limited communication and calculations possibilities and limited energy storage. Therefore, for proper functioning, they need optimized location of nodes of a WSN [1] and special communication protocols [2].

2. Reputation

Two concepts start playing an important role in modern applications of probability: trust and reputation. We can define *trust* as probability that a party will behave according to our expectation, honestly, in a specific situation. In this context, reputation can be defined as conditional probability of honest behaviour of a party where the condition is an available experience accumulated from past behaviour of the party. Such a definition can be generalized (e.g., to a vector-valued form, reflecting several aspects of reputation), but its probabilistic origin remains valid.

3. Reputation systems

To estimate reputation of a party or a service, we must define a reputation system, which enables collecting, exchanging, and processing appropriate information. We must also define a reputation measure, which lets us assigning reputation scores (e.g., probabilities) to events registered by the reputation systems. Existing reputation systems can be classified according to their functional properties (e.g., objective/subjective, centralized/distributed) and according to mathematical methods applied (probabilistic, fuzzy logic-based, deterministic), see [3].

Reputation systems found their application in practice, especially in web services, e.g., electronic auctions, e-shops, social networks, etc. There are also attempts to apply a reputation system for validating routing nodes in ad-hoc networks [4]. Such a tool was very useful for optimization of communication in Mobile Ad-hoc Networks under random disturbances.

4. Optimal sensors' location

Permanent structure's monitoring enables immediate and effective detection of its failure or its anomalous behaviour. It is especially important for proper functioning of intelligent structures and mechanisms and for their safety. In the literature one can find methods of optimization of sensors'

location using different criteria [5]. One of most often used criterion comes from information theory [6]; it is extensively used in problems of practical interest [7], [8], [9].

Application of WSNs for environmental and structural monitoring not only makes it possible permanent observation of working structures but also increases functionality of the monitoring system and decreases its overall cost. However, designing WSN-based monitoring system, one must take into account additionally (except of usual engineering constraints) specific restrictions connected with low energy broadcasting in WSN. Thus, in such a case the optimal location of sensors should take into account, both, conditions connected with structure's behaviour (finding the most "informative" measurement points) and environmental conditions determined by random disturbances of transmission of a signal measured, depending on external electromagnetic fields and structural barriers, see [10].

Thus, optimization of sensors' locations in a WSN for structural monitoring needs taking into account criteria of quite different nature. Firstly, we must optimize communication in an ad-hoc network (a quality of transmission, the network performance and lifetime of the sensors) with an additional constraint of a limited communication range of sensors. Secondly, we should choose such locations of sensors that give best measurements for specific engineering purposes. A natural solution seems to be application of a multi-criteria optimization system, which unifies very different measures of quality to a single decision criterion. The proposed application of a reputation system dedicated to the WSN dodges the problem of different optimality criteria for communication and for measurements. In these both cases, we measure quality with probability of certain events, so consolidation of the obtained results can be made on grounds of the probability theory. Moreover, such an approach makes it possible to construct an adaptive optimization system consistent to the reputation-based optimality criterion. Statistical data collected during measurements are a good source for reputation estimations of all events required to choose optimal locations of sensors in the WSN designed for most structural and environmental engineering problems.

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APPROXIMATE AND EXACT SOLUTIONS OF THE FIRST-PASSAGE PROBLEM FOR STOCHASTIC OSCILLATORS

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1. Introduction and motivation

In problems that cover a wide range of engineering applications, a reference mode of the system operation is associated with motion within an admissible domain G; exit from this domain can be in some sense catastrophic. Examples are the loss of integrity and the destruction of structures, the loss of stability, etc. The mean time to escape from a region of desired operations is one of the most basic reliability measures in stochastic dynamics.

Formally, the mean escape time $\mathbf{E}\tau$ can be found as a solution to the boundary problem for a relevant Fokker-Plank equation [1]. Whilst there are a small number of cases for which closed-form solutions are available, in most cases one must attempt numerical approximation, and so one is limited to only low dimensional problems. Even putting aside the restriction of numerical methods to low dimensions, one would prefer an explicit formula for $\mathbf{E}\tau$ since they have many other uses. In particular, beyond simply identifying the rate of decay, an analytic expression for $\mathbf{E}\tau$ can be used to characterize the most likely way to escape and/or to choose an efficient control strategy.

Over last decades, much attention has been given to weakly perturbed systems with weak dissipation, e.g. [2] – [4]. The purpose of this paper is to discuss recent results concerning explicit solutions of the first-passage problem for two opposite classes of stochastic oscillators, a multidimensional non-dissipative oscillator with non-small additive noise and a weakly perturbed oscillator with significant dissipation.

2. Stochastic models

As a first model, we consider a multidimensional non-dissipative oscillatory system excited by additive white noise. The equations of motion are written in the Lagrangian form

$$\frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{q}} - \frac{\partial L(q,\dot{q})}{\partial q} = \sigma \dot{w}(t)\,,\ q,\ \dot{q}\ \in G \in R^{2n} \tag{1}$$

where $q \in R^n$ is the vector of generalized coordinates; $w(t)$ is standard Wiener process in R^m; $L(q,\dot{q}) = T(q,\dot{q}) - U(q)$ is the Lagrangian of the system, $T(q,\dot{q})$ is the kinetic energy, $U(q)$ is the potential energy. The diffusion matrix σ yields a symmetric positive definite matrix $A = \sigma\sigma^{\mathrm{T}}$. In general, the reference domain G is considered as a connected open bounded set in R^{2n} with smooth boundary Γ and compact closure $\bar{G}$.

Escape time for system (1) is defined as the time needed to reach a critical level of energy H^* from the initial state with the energy H^0. This implies that

$$G\colon \{H^0 \le H < H^*\}\ ,\ \Gamma\colon \{H = H^*\} \tag{2}$$

In sharp contrast to the great majority of stochastic problems, for the non-dissipative oscillator (1) one can obtain a precise analytic solution of the Fokker-Plank equation. As shown in [5], the mean time to escape from the domain (2) is defined as

(3) $$\mathbf{E}\tau = \frac{H^* - H^0}{\mathbf{Tr}A}.$$

In order to take proper account of the effect of dissipation, we consider an opposing model. The equations of motion are written as

(4) $$\frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{q}} - \frac{\partial L(q,\dot{q})}{\partial q} + B(q,\dot{q})\dot{q} = \varepsilon\sigma(q,\dot{q})\dot{w}(t),\ q,\ \dot{q} \in G \in R^{2n},$$

where $B(q,\dot{q})$ is the matrix of dissipation forces; the small parameter $\varepsilon > 0$ implies the weak effect of noise compared to the effect of potential and dissipation forces. The matrices $B(q,\dot{q})$ and $A(q,\dot{q}) = \sigma(q,\dot{q})\sigma^{\mathrm{T}}(q,\dot{q})$ are assumed to be symmetric positive definite in $\bar{G}$.

Taking the noiseless system asymptotically stable, we construct a closed-form logarithmic asymptotics of $\mathbf{E}\tau^{\varepsilon}$ as $\varepsilon \to 0$ by means of the large deviations techniques [2], [3]. It has been shown [6] that the asymptotic estimate can be represented as a sum of two terms associated with the kinetic and potential energy, respectively. The first term can be found explicitly; the second term satisfies a linear PDE. The explicit solution of the latter equation can be obtained for several classes of systems.

Finally, we use the solution of the above problem to develop a control strategy ensuring a noise-independent escape rate in the controlled system. This result is exploited to design a stabilizing control for a gimbal suspensions gyroscope.

3. Acknowledgement

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THE INFLUENCE OF STATISTICAL FREQUENCY SCATTER ON PEDESTRIAN DESIGN LOADS FOR FOOTBRIDGES

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1. Background

Many modern footbridges are light structures in the sense that the non-permanent load is rather large compared to the permanent load, constituted by the weight of the bridge. In recent years footbridges have become longer, and more slender and flexible designs have been adopted. As a result, dynamic excitation – typically from the pedestrians using the bridge – have become a major design issue. A typical design procedure – originally introduced in [1] and later refined in [2] and [3] – consists in a Fourier expansion of typical footfall records, followed by evaluation of the response to the harmonic component closest to the base frequency of the bridge. The corresponding response acceleration of the original structure is of the form

$$\ddot{\mathbf{u}} = \omega_j^2 r_j \mathbf{u}_j = \frac{q_j}{2\varsigma_j m_j}\mathbf{u}_j$$

In this formula $\ddot{\mathbf{u}}$ is the physical acceleration, and r_j is the amplitude of the mode-shape vector $\mathbf{u}_j$ with angular frequency ω_j. The result corresponds to a simple resonance, where the load is represented by the modal load intensity q_j, while the denominator is the product of the modal damping ratio ς_j and the modal mass m_j. This formulation leads to a very simple design procedure, because it has been shown [4] that the resonant response of a mode shape with a tuned mass damper can be represented by an equivalent damping ratio, which is half the damping ratio applied in the tuned mass damper.

The advantage of the above argument is its simplicity in design, see e.g. [3]. However, it will often be a rather conservative design procedure, because it is based on the assumption that the excitation is concentrated in a very narrow frequency interval around the natural frequency of the structure. Experimental measurements of the loading from walking pedestrians indicates that the frequency has a coefficient of variation around 0.06-0.09 [5,6]. Thus, the spread of the loading frequency is much larger than the width of the resonance peak of the original structure. This means that the severity of the loading of the original bridge without additional tuned mass absorber(s) is overestimated, and that the response of the bridge after installation may be inaccurately determined.

2. Stochastic design model

The literature contains numerous formulae giving slightly different ‘optimal’ damping and frequency parameters for tuned mass absorbers. However, it has been demonstrated via analytical solutions for the extreme case of white noise excitation, that calibration of the tuned mass absorber for a resonant load gives the same response to within a fraction of a percent [7]. This observation leads to a simple procedure for evaluating the effect of frequency spreading on the bridge response – both in the original state and after installation of a tuned mass absorber.

The loading including a representative frequency spread can be represented by a simple rational function centred around the mean footfall frequency and with an appropriate coefficient of variation. When extended symmetrically to include negative frequencies this leads to a spectral density that can be generated from white noise by a second order filter. The system, consisting of

the structure – here represented by an idealized single-degree-of-freedom system – and the tuned mass, is represented by a coupled two-degree-of-freedom system. In total the system including the tuned mass absorber can be described in terms of six state-variables driven by a white noise process. As mentioned above the tuned mass absorber parameters can be calibrated as if the load were harmonic, and thus a parametric analysis of the effect of the frequency characteristics of the loading can be carried out by computing the covariances of the system by use of Lyapunov's equation. The paper will show representative results and discuss the influence on realistic design situations for footbridges.

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FROM MICROSCOPIC TO MACROSCOPIC DESCRIPTIONS OF COMPLEX SYSTEMS

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1. General statement

The general approach that relates **microscopic** and **nonlocal** models with those at the **macroscopic** scale is considered. The macroscopic models are in terms of a system of reaction-diffusion equations. The micro-scale level is defined by a large number of interacting entities, and is in terms of a Markov jump process and related linear evolution equations. The intermediate scale (meso-scale) refers to the description of test-entities and is given in terms of nonlinear Boltzmann--type equations. Mathematical relationships between these three possible descriptions are proved and explicit error estimates are given.

2. Details

Usually the description of complex systems (e.g. biological populations) is carried out on a macroscopic level of interacting subpopulations within the system. Such an approach can be related to deterministic reaction-diffusion equations. They describe the (deterministic) evolution of densities of subpopulations of the system rather than the individual entities.

However, in many cases the descriptions on a micro-scale or a meso-scale of interacting entities (particles, cells, individuals, ...) seem to be more appropriate. The important feature of the microscopic level can be a nonlocal way of interactions: one entity may interact with another one even if the distance between them is not negligible.

As a prototype of the mathematical setting and relationships between three possible scales of description: micro, meso and macro can be kinetic theory of rarefied gases. There is however an important difference: in the case of general (e.g. biological) systems a basic microscopic theory, as the Newton Laws in kinetic theory case, it is not available. Therefore it is reasonable to apply the following strategy. One may start with the deterministic macroscopic model for which the identification of parameters by an experiment is easier. Then one may provide the theoretical framework for modelling at the microscopic scale in such a way that the corresponding models at the macro- and micro-scales are asymptotically equivalent, i.e. the solutions are close each to other in a properly chosen norm. Then, if the parameters of the microscopic model are suitably chosen, one may hope that it covers not only macroscopic behaviour of the system in question, but also some of its microscopic features. The microscopic model by its nature can be richer and can describe a larger variety of phenomena.

We review a general conceptual framework for the program ([1]-[4]) of finding possible transitions between the different levels of description i.e.

(Mi) at the level of interacting entities (***micro-scale***),

(Me) at the level of the statistical description of a test-entity (***meso-scale***),

(Ma) at the level of densities of subpopulations (***macro-scale***).

The levels **(Mi)** and **(Me)** are of the nonlocal character.

In mathematical terms we are interested in the links between the following mathematical suitable defined structures:

(Mi) the micro-scale of stochastically interacting entities, in terms of jump Markov processes, that lead to continuous (linear) stochastic semigroups;

(Me) the meso-scale of statistical entities, in terms of continuous nonlinear semigroups related to the solutions of nonlinear Boltzmann-type nonlocal kinetic equations;

(Ma) the macro-scale of densities of interacting entities, in terms of dynamical systems related to nonlinear reaction-diffusion equations.

In Ref. [1]-[4] such a conceptual framework was developed for various situations of biological interest. In particular, Ref. [1] deals with the mathematical theory for a large class of reaction-diffusion systems (with small diffusion). Ref. [2] shows that the theory can be generalised to take into account reaction-diffusion-chemotaxis systems (i.e. reaction-diffusion equations with a chemotaxis-type term).

These methods may lead to new and more accurate modelling of complex processes.

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STOCHASTICITY AND TIME DELAYS IN GENE EXPRESSION AND EVOLUTIONARY GAME THEORY

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Regulation of gene expression is a chemical process involving many coupled elementary chemical reactions modelled usually by systems of differential equations describing time evolution of molecular concentrations. However, due to low numbers of molecules involved in gene expression processes, random fluctuations may play a significant role. To describe stochastic effects in biochemical reactions taking place in living cells, various birth and death processes were studied.

We will review a simple model of protein production which can be completely solved, that is one can obtain analytical expressions for the expected value and the variance of the number of protein molecules [1]. Then we will discuss specific models of mRNA- and protein-regulated networks. In particular, we will discuss contributions of regulatory factors to gene expression noise in four basic mechanisms of negative gene expression control: 1) transcriptional regulation by a protein repressor,
2) translational repression by a protein, 3) transcriptional repression by RNA, and 4), RNA interference with the translation. Our results show that translational repression results in a higher noise than repression on the promoter level [2].

In the standard birth and death processes, products of various reactions appear or degrade immediately after corresponding reactions are triggered. However, many such reactions take a considerable amount of time. Therefore to describe them we have to introduce models with time delays. Reactions with delays are of two kinds: non-consuming and consuming. Reactants of unfinished consuming reactions cannot participate in new reactions, reactants of non-consuming reactions can participate in new reactions. We will discuss simple models of gene expression with time delays. We will analyze both kinetic rate equations and corresponding birth and death processes with both types of time delays. Many kinetic rate equations with non-consuming reactions undergo the Hopf bifurcation when the delay increases and crosses a critical value. For small time delays the system evolves into its stationary state with damped oscillations observed in transient states. We will show that such effects are not present in the case of consuming reactions where for all values of time delay the unique stationary state is asymptotically stable. In the stochastic models corresponding to deterministic rate equations, the variance of the number of protein molecules and autocorrelation functions will be calculated analytically. To deal with more complex models, we will develop a small delay approximation. We compare our results with those obtained earlier in [3].

As we have learned from the above discussion, the effect of time delays on the stability of various dynamical systems depends very much on physical origins of delays. Similar situation appears on a larger time scale in population models. The evolution of populations can be often described within game-theoretic models [4,5]. The fundamental notion here is that of an evolutionarily stable strategy. If everybody plays such a strategy, then the small number of mutants playing a different strategy is eliminated from the population. The dynamical interpretation of the evolutionarily stable strategy is provided by a system of differential or difference equations, the so-called replicator equations. They describe the time-evolution of frequencies of strategies. It is known that evolutionarily stable strategies are asymptotically stable stationary points of such dynamics.

Stochastic stability of evolutionarily stable strategies was analyzed in many papers, see for example [6,7] and a recent review [8].

Various aspects of time delays in replicator dynamics in evolutionary game theory were discussed recently in [8,9]. Two specific models of two-player games with two strategies and a unique mixed evolutionarily stable strategy were analyzed. In the social-type model, players imitate opponents taking into account average payoffs of games played some units of time ago. In the biological-type model, new players are born from parents who played in the past. We have shown in [9] that in the first type of dynamics, the unique mixed evolutionarily stable strategy is asymptotically stable for small time delays and becomes unstable for large ones when the population oscillates around its stationary state. In the second type of dynamics, however, the evolutionarily stable strategy is asymptotically stable for any time delay.

The interplay of stochasticity and time delays in evolutionary game theory and other population dynamics models as well as in various models of genetic regulatory networks is a subject of a current intensive research.

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REANALYSIS TECHNIQUES IN STOCHASTIC MECHANICS

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Abstract

In various structural design and optimization problems it is necessary to repeatedly evaluate the response of the structure for numerous modified designs. Specifically, mechanical and geometrical parameters may change and eventually structural components can be added or deleted leading also to a change in the number of the degrees of freedom in the pertinent finite element model. Since, in general, the response cannot be expressed explicitly in terms of the structure properties, any structural modification requires a considerable computational effort. These computational mechanics problematic are to the base of the increasing interest in the reanalysis techniques in last two decades. Indeed, the aim of the reanalysis is the valuation of the structural response of modified systems using the results relative to the original structure, called as reference structure, so reducing the computational effort. In this regard the reanalysis techniques are classified as topological or non-topological if the modifications lead to a change of the degrees of freedom of the system or not.

Therefore, the reanalysis is able to cope with both static and dynamic response evaluation. A review of the static response reanalysis techniques can be found in [1] and more recently in [2], in which it is shown that the most common reanalysis methods, namely: the Combined Approximation (CA); the Theorems of Structural Variations (TSV) and the Virtual Distortion Method (VDM), can be derived from the Sherman-Morrison-Woodbury formula [3,4]. The previous quoted methods can be also adopted for topological modifications [5]. A review of the earliest contribution in this topic can be found in [6-7].

Most of the existing reanalysis methods in dynamics are concentrated on resolving the modal problem, in which only modifications to eigenvalues and eigenmodes are considered. This problem is solved quasi-statically in the frequency domain (no dependence on time is investigated). A review of some eigenproblem reanalysis methods can be found in [7-8]. Recently the authors, proposed a method for the dynamic response reanalysis in the time domain [9-10], the method evaluates the structural response by the dynamic modification approach [11] without solving any eigenproblem. The method can be adopted to evaluate both deterministic and stochastic response.

In the framework of stochastic mechanics two problems are the object of this study. In the first one the reanalysis technique herein proposed is used for Monte Carlo simulations of structures with uncertain parameters. Lastly, the presented approach is also applied for the random response of linear systems subjected to random loadings. In the former problem several analyses of slightly modified systems are required. Indeed for systems with uncertain parameters, if the Monte Carlo simulation is adopted, the computational effort is quite onerous. In this regard, the main steps involved in a Monte Carlo study require the simulation of a set of random variable from a theoretical distribution of structural parameters, the deterministic analysis of the response for each set of variables, and the evaluation of the response statistics determined by repeating several times the deterministic analysis for each individual new simulation. Clearly, to avoid the very onerous required repeated analyses, the deterministic dynamic response reanalysis is computationally very useful. In this regard, the Dynamic Modification Method (DMM) proposed in [11] is extensively used and resorted to cope with the dynamic response reanalysis in time domain. According to this method all the dynamic modifications are assumed pseudo-forces and the response of the modified

structure is retrieved starting from the knowledge of the transition matrix and the eigenvectors of the original structure.

Moreover, via this approach the response of nonclassically damped system is determined without the evaluation of complex quantities. Lastly, the present approach is also applied to the random response of linear systems subjected to random loadings. For simplicity's sake only the case of non topological modifications is considered..

The numerical results show the accuracy and the computational efficiency of the described approach to the analysis of multi-degrees-of-freedom (MDoF) systems. Remarkably, it is also shown that the approach is computationally very effective, in order to show this the CPU reduction time is evaluated.

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FIRST PASSAGE FAILURE OF A LINEAR OSCILLATOR UNDER ADDITIVE AND MULTIPLICATIVE RANDOM EXCITATIONS

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1. INTRODUCTION

The focus of our paper is the following linear oscillator under both additive and multiplicative random excitations:

$$\ddot{X} + \omega_0[2\zeta + W_2(t)]\dot{X} + \omega_0^2[1 + W_1(t)]X = W_3(t), \tag{1}$$

where $W_j(t))$, $j = 1, 2, 3$, are wideband stationary processes with zero mean values. This model was studied by Ariaratnam and Tam [1] under the assumption that ζ is of order ϵ and the $W_j(t))$ are of order $\sqrt{\epsilon}$, where ϵ is a small parameter. By applying the stochastic averaging procedure, it was argued that the amplitude process $A(t) = \left(X^2 + \dot{X}^2/\omega_0^2\right)^{1/2}$ is approximately a Markov diffusion process governed by the (Itô) stochastic differential equation (SDE)

$$dA = m(A)dt + \sigma(A)dB(t)\,. \tag{2}$$

The drift coefficient $m(A)$ and the diffusion coefficient $\sigma(A)$ are given by the equations,

$$m(A) = -\alpha A + \frac{\delta}{2A}\,, \tag{3}$$

$$\sigma(A) = (\gamma A^2 + \delta)^{1/2}\,, \tag{4}$$

in which

$$\alpha = \zeta\omega_0 - \frac{\pi\omega_0^2}{8}\left[2\Phi_{22}(0) + 3\Phi_{22}(2\omega_0) + 3\Phi_{11}(2\omega_0) - 6\Psi_{12}(2\omega_0)\right], \tag{5}$$

$$\delta = \frac{\pi}{\omega_0^2}\Phi_{33}(\omega_0)\,, \tag{6}$$

$$\gamma = \frac{\pi\omega_0^2}{4}\left[2\Phi_{22}(0) + \Phi_{22}(2\omega_0) + \Phi_{11}(2\omega_0) + 2\Psi_{12}(2\omega_0)\right], \tag{7}$$

and

$$\Phi_{ij}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{E}[W_i(t)W_j(t+\tau)]\,\cos(\omega\tau)\,d\tau\,,\;\; i,j = 1,2,3\,, \tag{8}$$

$$\Psi_{ij}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{E}[W_i(t)W_j(t+\tau)]\,\sin(\omega\tau)\,d\tau\,,\;\; i,j = 1,2,3\,. \tag{9}$$

Ariaratnam and Tam [1] showed that the expected time $\langle T_f \rangle$ to first failure of the amplitude process $A(t)$ is given by the formulas

$$\langle T_f \rangle = \frac{1}{\eta\gamma} \int_{a_0}^{a_c} \frac{1}{u} \big[\big(1 + \frac{\gamma}{\delta} u^2\big)^{\eta} - 1 \big] \, du \,, \;\; \eta = \frac{\alpha}{\gamma} + \frac{1}{2} \neq 0 \tag{10}$$

$$\langle T_f \rangle = \frac{1}{\gamma} \int_{a_0}^{a_c} \frac{1}{u} \ln \big(1 + \frac{\gamma}{\delta} u^2\big) \, du \,, \;\; \eta = 0 \tag{11}$$

Here a_0 denotes the initial condition and a_c the critical level ($a_0 < a_c$). This approach would usually represent an approximation in the sense that failure for the original problem would typically be when $X(t)$ exceeds a critical threshold x_c. An approximate solution for this is obtained by studying the exceedance of $a_c = x_c$ by the amplitude process $A(t)$.

Using numerical path integration we have calculated the reliability function associated with the linear oscillator model in Eq. (1) for a range of parameter values. Since this can be done for any choice of parameter values, it provides a means of studying the limitations of the amplitude diffusion model adopted in [1], and thereby also the limitations of stochastic averaging.

The reliability is defined in terms of the displacement response process $X(t)$ in the following manner, assuming that all events are well defined,

$$R(\, T \,|\, x_0, 0, t_0) = \text{Prob}\{x_l < X(t) < x_c;\, t_0 < t \leq T | X(t_0) = x_0,\, Y(t_0) = 0\}, \tag{12}$$

where x_l, x_c are the lower and upper threshold levels defining the safe domain of operation. It has been shown [2] that

$$R(\, T \,|\, x_0, 0, t_0) \approx \int_{-\infty}^{\infty} \int_{x_l}^{x_c} \cdots \int_{-\infty}^{\infty} \int_{x_l}^{x_c} \prod_{j=1}^{n} p(z_j, t_j | z_{j-1}, t_{j-1}) \, dz_1 \cdots dz_n, \tag{13}$$

which is the path integration formulation of the reliability problem. Here, $p(z, t|z', t')$ denotes the transition probability density function of the state space vector process $Z(t) = (X(t), Y(t))^T = (X(t), \dot{X}(t))^T$, and $t_j = t_0 + j\Delta t$, $j = 1, \ldots, n$, and $\Delta t = (T - t_0)/n$.

The complementary probability distribution of the time to failure T_f, i.e. the first passage time, is given by the reliability function. The mean time to failure $\langle\, T_e \,\rangle$ can thus be calculated by the equation

$$\langle\, T_f \,\rangle = \int_0^{\infty} R(\, \tau \,|\, x_0, 0, t_0) \, d\tau \tag{14}$$

The results obtained by Eqs. (10), (11) and from Eq. (14) by path integration, can then be compared. This will shed some light on the performance of stochastic averaging methods.

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Multiscale dynamics and information: some mathematical challenges

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This talk outlines multiscale problems which combine techniques of model reduction and filtering. Multiple time scales occur in models throughout the sciences and engineering, where the rates of change of different variables differ by orders of magnitude. The basis of this work is a collection of limit theories for stochastic processes which model dynamical systems with multiple time scales. When the rates of change of different variables differ by orders of magnitude, efficient data assimilation can be accomplished by constructing nonlinear filtering equations for the coarse-grained signal. We consider the conditional law of the coarse-grained signal given the observations. In particular, we study how scaling interacts with filtering via stochastic averaging. We combine our study of stochastic dimensional reduction and nonlinear filtering to provide a rigorous framework for identifying and simulating filters which are specifically adapted to the complexities of the underlying multi-scale dynamical system.

State estimation of random dynamical systems with noisy observations has been an important problem in many areas of science and engineering. Since the true state is usually hidden and evolves according to its own dynamics, the objective is to get an optimal estimation of the true state via noisy observations. The theory of filtering attempts to give a recursive procedure for estimating an evolving signal or state from a noisy observation process. When a system has several different scales, one can seek reduced order models whose essential dynamics describe the evolution of the full system with small number of the state variables. In systems subject to both bifurcations and noise (which form one of the main components of this talk), various *singular perturbations* problems must be understood. To this end, reduced models often provide qualitatively accurate and computationally feasible descriptions. The lower-dimensional model is strictly valid only in the limit of infinitesimally small noise. Nonetheless, the stochastically averaged model should provide qualitatively correct results and be potentially helpful in developing inexpensive lower-dimensional computational models.

The first objective of this talk is concerned with certain methods of *dimensional reduction* of nonlinear systems with symmetries and small noise [1]. In the presence of a separation of scales, where the noise is asymptotically small, one exploits symmetries to use recent mathematical results concerning *stochastic averaging* to find an appropriate lower-dimensional description of the system. The unique features of the problem are interactions between bifurcations, resonances, dissipation and random perturbations. Bifurcations are where small changes in a system result in large changes in the structure of the fast orbits [2]. The subtleties of the interaction between these effects will lead to new and novel analytical techniques. Hence, we are developing techniques of stochastic dimensional reduction to find a simpler model which predicts or captures relevant dynamics of the system [3]. One of the preeminent modern frameworks for considering convergence of the laws of Markov processes is that of the *martingale problem*, which we will use in deriving the coarse-grained dynamics [4].

The second objective of this talk is to develop, with mathematical rigor, a lower - dimensional nonlinear filter by combining two ingredients, namely, stochastic dimensional reduction discussed above and nonlinear filtering. We find a reduced nonlinear filtering problem when the

system dimension can be reduced via homogenization. We approximate the complex original nonlinear filtering equation by simpler ones with a quantifiable error. This is an extension of our previous work [5,6] to a more realistic setting where the observation depends on both slow and fast variables.

In this talk we derive a low-dimensional filtering equation, that determines conditional law of a plant, in a multi-scale environment given the observations. This talk is less concerned with specific applications and more focused on some of the theoretical aspects that deal with reduced dimensional nonlinear filters. In particular, we showed the efficient utilisation of the low-dimensional models of the signal to develop a low-dimensional filtering equation. We achieved this through the framework of homogenisation theory which enables us to average out the effects of the fast variables. Reduced models can be used in place of the original complex models, either for simulation and prediction or real-time control. To this end, reduced models often provide qualitatively accurate and computationally feasible descriptions.

In conclusion, we are interested in something of a "nongeneric" system which is not amenable to direct probabilistic asymptotic analysis, but which has been overlooked in the homogenisation literature. Another aspect of note is that our interest is specifically dimensional reduction of the plant, not homogenisation. Our analysis will hinge upon an application of the tools of stochastic averaging to a study of the Zakai equation.

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STOCHASTIC AND CHAOTIC ANALYSIS OF SHALLOW CABLES DUE TO CHORD LENGTH ELONGATIONS

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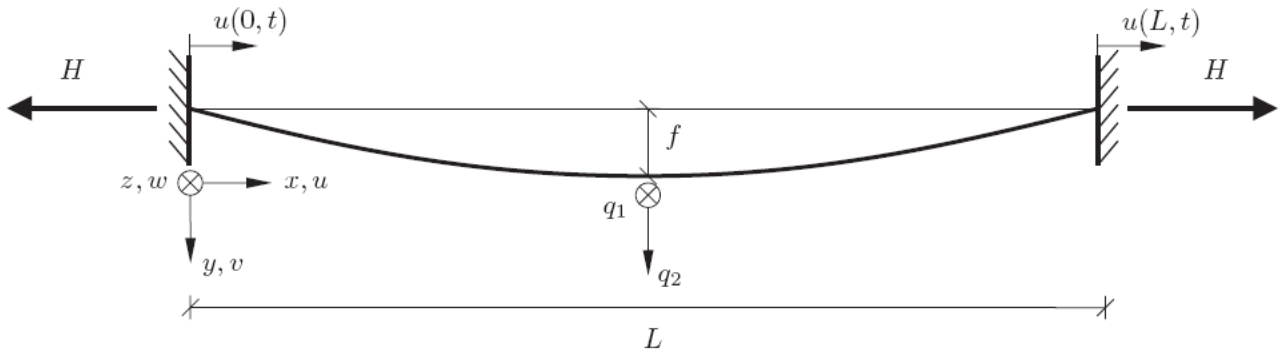

Figure 1. Cable in static equilibrium configuration.

Cable systems are of great interest in a wide range of applications in civil engineering to supply both support and stiffness to large structures. Typically, cables are used as support of cable-stayed bridges, masts and TV-towers are characterized by a sag-tochord-length ratio below say 0.01, which means that the angular eigenfrequencies for the in-plane eigenvibrations $\omega_2,\ \omega_4,\ldots,$ and the out-of-plane eigenvibrations $\omega_1,\omega_3,\ldots$ are pairwise close. With reference to the coordinate system defined in figure 1 the components of the support point motion in the *y* and *z* directions merely introduce additive load terms in the modal equations of motion of the cable, whereas the chord elongation $u(L,t)-u(0,t)$ along the x-axis causes additional parametric loading terms in the modal equations of motion, which may cause significant subharmonic and superharmonic responses. The chord elongation is conveniently described by the following non-dimensional parameter of the magnitude 1

$$e(t)=\frac{EA}{HL}\big(u(L,t)-u(0,t)\big) \tag{1}$$

where EA/L denotes the axial stiffness and H is the pre-stressing force. Even though the excitation only affects the in-plane motion, stable out-of-plane displacements may be brought forward by non-linear couplings in both harmonic, subharmonic and superharmonic responses.

When the chord elongation and hence $e(t)$ is harmonically varying with the e_0 and the angular frequencies ω stable stationary periodic motions exist for specific frequency ratios ω/ω_1. Figure 2a shows the trajectory of the midpoint of the cable for subharmonic response of order 2 for $\omega/\omega_1=2$. As seen, the in-plane modal coordinate $q_2(t)$ is rather small and harmonically moving with the same frequency as the excitation, whereas the out-of-plane coordinate $q_1(t)$ is large at subharmonic resonance with a frequency equal to half the excitation frequency. The stable trajectory is brought forward by a phase locking between the in-plane and out-of-plane components. However, in reality the chord elongation is narrow banded stochastic rather than harmonic varying, driven by the narrow-banded random response of the supported structure. In this case the subharmonic response of the cable changes dramatically, qualitatively and quantitatively, no matter

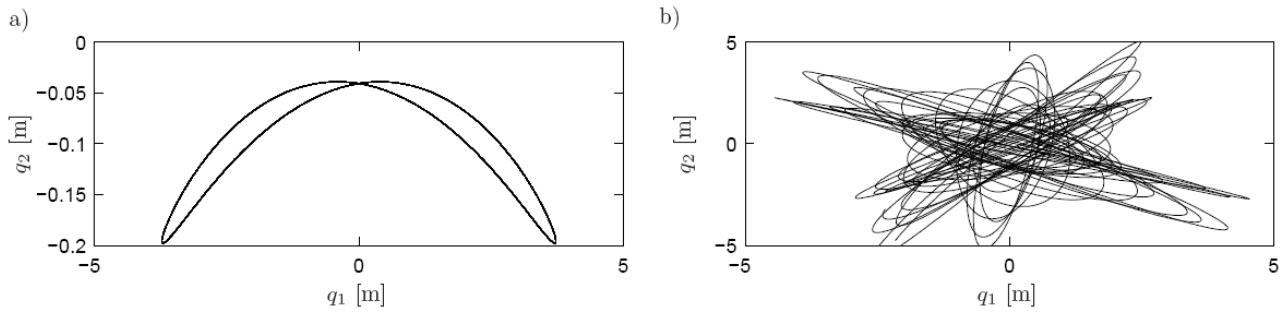

Figure 2. Subharmonic response of order 2. a) Harmonic varying chord elongation. b) Stochastic chord elongation.

how small the bandwidth of the excitation is. As shown in figure 2b, the in-plane and the out-of-plane components are coupled forming a elliptic like trajectory with slowly varying inclination and magnitude of the semi-axes.

The response is defined as chaotic with probability one, if two realizations with close initial values exposed to the same but arbitrary realization of the chord elongation process deviate exponentially with time. The exponential growth rate is measured by the Lyapunov exponent, which here is estimated numerically by the algorithm of Wolf et al. Chaotic behaviour occurs for sufficiently large standard deviation of the excitation process. In the paper stochastic chaotic response is investigated for subharmonic response of order two, and superharmonic response of the orders 3/2 and 2. It is demonstrated by means of Monte Carlo simulation that in all the indicated cases the tendency to stochastic chaotic behaviour is increased for increased standard deviation and increased bandwidth of the excitation process is increased. Further, the magnitude of the out-of-plane displacement is also dependent on the bandwidth, and ceases completely above a certain critical bandwidth parameter. Finally, it is demonstrated that stochastic excitation processes with the same auto-spectral density function, but different higher moments provide qualitatively identical stochastic ordered and chaotic responses, i.e. the dramatic influence of the stochastic excitation on the response is basically caused by the second order moments.

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FATIGUE LIFETIME PREDICTIONS IN METALLIC STRUCTURES USING LIMITED NUMBER OF VIBRATION MEASUREMENTS

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1. Objective

This work deals with the problem of estimating damage accumulation and predicting the lifetime due to fatigue in the entire body of a metallic structure using output-only vibration measurements from a sensor network installed at a limited number of locations in a structure. The objective is to formulate the problem, present the methods and assumptions under which effective fatigue predictions can be accomplished, outline the main issues and factors that affect the accuracy of the predictions, as well as illustrate the effectiveness of the methods with selected applications using simulated measurements.

2. Introduction

Damage accumulation due to high-cycle fatigue is an important safety-related issue in metallic structures. Methods exist to estimate the damage accumulation at a point in a structure using the response time histories of the components of the stress sensor at the corresponding point. One such method integrates experimentally obtained S-N fatigue curves for damage prediction, Miner linear damage accumulation law accounting for arbitrary stress response time histories, cycle counting per stress level methods in stress response time histories, and methods for handling multi-axial stress states at a point. These methods can be applied to any point in the structure and construct the complete fatigue map of the entire structure, provided that the stress response time histories at all desirable points are available.

The characteristics of the stress response time histories at a point in a structure can readily be predicted by using a model of the structure (e.g. a finite element model) and the actual excitation time histories. However, for most structures, the excitation time histories are neither available nor can be conveniently measured by a system of sensors, while simplified representations of excitation models often used for design purposes do not reflect the actual excitation conditions during various phases of operation of the structure. In addition, the models of the structure may lack the desirable accuracy due to model error. Alternatively, the stress response time histories at a structure can be readily inferred using measurements obtained by placing strain rosettes. Even in this case, there are significant limitations in predicting the stress time histories in the entire body of the structure since the number of sensors in a sensor network is usually very limited and cannot cover all points or critical structural locations. Moreover, structural locations may not be all approachable to place sensors (e.g. hard to reach structural locations in large extended structures, submerged structures, heated structural components, internal points in a structure).

This work concentrates mainly on proposing methods for predicting the characteristics of the stress response time histories in the entire body of the structure using measurements collected from a sensory system placed at limited number of structural locations. Such predictions can be employed for estimating damage accumulation maps due to fatigue. The excitation in the structure during its operation is considered to be unknown. The main assumptions for the proposed predictions are that the structure is linear, the responses are realizations of a stochastic stationary process, and the unmeasured excitations can be modelled by stationary stochastic processes.

3. Methodology

Available frequency domain stochastic fatigue methods based on Miner's damage rule, S-N fatigue cycle curves, and Dirlik's probability distribution of the stress range are used to predict the expected fatigue accumulation of the structure in terms of the power spectral density (PSD) of the components of the stress tensor process [1,2]. Thus, the problem is reduced to estimating the PSDs of the stress components at unmeasured locations using the available measured response time histories at limited locations of the structure. These PSDs of stresses are estimated by using stochastic representations of the excitation models, Kalman filter techniques [3] or kriging methods. The accuracy of the predictions in such methods under unknown stochastic excitations depend highly on the accuracy of the stochastic process models used to represent the uncertain temporal and spatial characteristics of the excitation time histories, as well as the accuracy of the models used to represent the structural behaviour. Model identification methods are proposed to estimate, based on the vibration measurements, reliable stochastic models of the uncertain excitations, as well as update the structural models. In particular, it is demonstrated that optimal sensor configuration strategies are useful tools for improving predictions.

4. Results and Conclusions

The proposed formulation is demonstrated using simulated measurements from (a) an N-DOF spring-mass chain model arising from structures that consist of members with uniaxial stress states and (b) a three-dimensional structure that is modelled by shell/plate elements with bi-axial stress states. The factors that affect the accuracy of the methodology are investigated by comparing the fatigue accumulation results in the entire structure predicted by the proposed methodology with reference fatigue accumulation results. It is shown that the accuracy of the proposed method for fatigue predictions in the entire body of the structure depends on the number and location of sensors in the structures, the number of modes contributing in the dynamics of the structure, the size of the model error and measurement error, and the accuracy of the stochastic excitation models.

The proposed methodology can be used to construct fatigue damage accumulation and lifetime prediction maps consistent with the actual operational conditions provided by a monitoring system. In particular, the stochastic excitation models identified from measurements under various operational states, can also be used with the stochastic fatigue method for lifetime prognosis purposes. The proposed method is useful for designing optimal fatigue-based maintenance strategies for metallic structures using structural vibration information collected from a sensor network.

5. Acknowledgements

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PROBABILISTIC RESPONSE OF NONLINEAR SYSTEMS VIA PI: NORMAL, POISSONIAN AND COMBINED WHITE NOISES

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1. Introduction

In Engineering field several actions are random in nature and then estimation of the response, of linear or nonlinear systems under these agencies, has to be developed through statistics. As well known random excitations may be simulated by two kinds of noises: normal and non-normal white noises. For systems under normal or non-normal white noise, the response statistics may be obtained by solving the Fokker-Plank Kolmogorov (FPK) equation or the kolmogorov-Feller equation, respectively. However, exact solutions of the partial differential equations governing the evolution of the response probability density function (PDF) are known only for very few cases. Alternatively, several approximate solutions techniques have been developed including variational methods, finite element method , and the path integration approach . The path integral solution (PIS) is an effective tool for evaluating the response in terms of PDF at each time instant [1-4].

2. General Features on Path Integration method

For summarizing the general features of the (PIS) method, it is better resorting to the case of a half oscillator driven by a white noise, whose equation of motion is given in the form:

$$(1) \qquad \begin{cases} \dot{\mathrm{X}}(t) = -\alpha \mathrm{X}(t) + f(\mathrm{X},t) + W(t) \\ \mathrm{X}(0) = \mathrm{X}_0 \end{cases}$$

where $f(\mathrm{X},t)$ is a deterministic nonlinear function of X(t) and t, α is a parameter that must be positive and *W(t)* is a white noise (zero-th order memory Markov process) and X_0 is the attendant initial condition (deterministic or random variable). The PIS allows us to capture the entire evolution of the response process in terms of PDF, having an assigned initial condition.

The starting point is the Chapman-Kolmogorov equation, that holds true, because of the Markovianity of the input and of the response:

$$(2) \qquad \mathrm{p}_{\mathrm{X}}(x, t+\tau) = \int_D \mathrm{p}_{\mathrm{X}}(x, t+\tau | \bar{x}, t) \mathrm{p}_{\mathrm{X}}(\bar{x}, t) \mathrm{d}\bar{x}$$

The latter, for τ small may be interpreted as a step-by-step procedure, that means, if we suppose that the PDF of the response at the generic time instant (t) is already known, we may evaluate the PDF of the response at the close time instant ($t+\tau$). Regarding the numerical implementation of the PIS method a computational domain *D* has to be selected. It is convenient to select a symmetrical computational domain with a maximum size equal to x_1, i.e. $-x_1 \le x \le x_1$. The size of the domain is identified by, first, running a Mont Carlo Simulation (MCS) with a low number of samples. Then, dividing the domain in a number $\mathrm{n_x}$ of intervals, for each grid point, the path integral from Eq. (2)

can be evaluated. By looking at this equation (2) it is apparent that the crucial point is to evaluate the kernel, where a conditional joint PDF is present. Considering the physical significance of this conditional joint PDF that is: from the whole trajectories of the response process $\mathrm{X}(t)$,we take those assuming in t the deterministic value $\bar{x}$, hereafter labelled $\bar{\mathrm{X}}(\rho)$ solution of the following differential equation:

$$(3) \qquad \begin{cases} \dot{\bar{\mathrm{X}}}(\rho) = -\alpha \bar{\mathrm{X}}(\rho) + f(\bar{\mathrm{X}},\rho) + W(t+\rho) \\ \bar{\mathrm{X}}(0) = \bar{x} \end{cases}$$

being $\bar{x}$ a deterministic initial condition and $0 \le \rho \le \tau$. It is worth stressing that: the CPDF of Eq.(1) coincides with the unconditional PDF of Eq.(3a) evaluated in τ , that is

$$(4) \qquad \mathrm{p}_{\mathrm{X}}(x, t+\tau \,|\, \bar{x}, t) = \mathrm{p}_{\bar{\mathrm{X}}}(x, \tau)$$

These are the general features of PIS, now the problem is to particularize the kernel and this will be dependent on the system and on the type of white noise.

It will be introduced an excursus to asses how versatile is PIS for evaluating probabilistic response of linear and nonlinear systems. Firstly the case of systems under normal white noise will be examined, secondly the case of systems under Poissonian white noise and lastly the case under combined noises that is normal and non-normal white noises acting simultaneously. The accuracy of the method is assessed using Monte Carlo simulation and the exact solution when the latter is available.

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SPACE TIME MODELLING OF SIGNIFICANT WAVE HEIGHTS VARIABILITY FOR FATIGUE ROUTING

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Significant wave height Hs is defined as four times standard deviation of the wave field and hence Hs squared is a measure of the average wave energy. The accumulated damage in ships can be related to the history of encountered significant wave heights. Fatigue routing means planning shipping so that the accumulated damage would be minimized.

In this talk we shall present a model for space time variability of significant wave heights over oceans. The model will be used to predict the fatigue damage caused by sea waves to structural details in a ship. The resulting distribution will be compared with the empirical one derived from an extensive measuring campaign.

GLOBAL SENSITIVITY OF STRUCTURAL VARIABILITY BY RANDOM SAMPLING

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1. GENERAL REMARKS

Mathematical models play an increasingly important role to simulate, study and predict future outcomes in every field of science. These models, generally, involve a large set of input variables (whose values are often imprecisely known). With problems of this type, it is important to understand the relationships between the input variables and the output. Sensitivity analysis (SA) allows one to study these relationships and identify the most significant factors or variables affecting the results of the model. The global SA methods can also be used for model calibration, model validation, decision making process, i.e., any process where it is useful to know which are the variables that mostly contribute to the output variability.

Let define the model $f : \Re^p \rightarrow \Re$ and $Y = f(\boldsymbol{X})$ that can also be considered as a "black box" where Y is the output of model and $\boldsymbol{X} = X_1, \ldots, X_p$ are p-independent inputs. The effect of the variance of an input or a group of input parameters contributes to the output variance of f are described using the so-called Sobol's indexes. For a model with p inputs, the number of Sobol's indices is $2^p - 1$; leading to an intractable number of indices as p increases. Thus, to express the overall output sensitivity to an input X_i, the total sensitivity index, $S_{T_i} = S_i + \sum_{j \neq i} S_{ij} + \sum_{j \neq i, k \neq i, j<k} S_{ijk} + \ldots$, can be used [1]. Recent global SA techniques take into account the entire range of variation of the inputs and aim to apportion the whole output uncertainty to the input factor uncertainties (see e.g [4]).

Working in a standard normal space (obtained by means of e.g. the Nataf transformation), where the factors $x_1, \ldots, x_p$ are independent standard normal random variables, it has been shown [5] that the total sensitivity indexes has the following upper bound:

$$S_i^{tot} \leq \frac{\nu_i}{D} = \frac{\int_{H^p} \left(\frac{\partial f}{\partial x_i}\right)^2 d\mathbf{x}}{D} \tag{1}$$

where D is the total variance of f, ν_i are functions similar to the Morris importance measures [2] and H^n represents the standard normal space of dimension p. The estimation of ν_i can be performed by simple Monte Carlo (independent random samples) simulation.

The major drawback of this approach is that it may be CPU time consuming, mainly because of the sampling method. In case the total sensitivity is evaluated for all p components, the cost to estimate p total sensitivity indices is $N_t = N(p + 1)$. It is well known that, for complex computer models, an accurate estimation of these indexes by the simple Monte Carlo method requires $N > 1000$. In complex industrial applications, this approach is intractable due to the CPU time cost of one model evaluation and the possible large number of input parameters.

2. PROPOSED APPROACH

The aim of this paper is to present an efficient Monte Carlo based approach for the estimation of the upper bound of S_i^{tot} where $n \ll N$ samples are required to estimate the (upper)

bound of each index. The main idea is to estimate the gradient components of a generic function by means of a Monte Carlo procedure along the points sampled from a Markov-chain. It has been shown [3] that adopting an orthogonal linear transformation it is possible to identify a new coordinate system where a relatively small subset of the variables contributes significantly to the gradient. Working in this transformed space only few samples, i.e. much less than the dimensionality of the problem, are required in order to estimate the gradient components, $\frac{\partial f}{\partial x_i}$. The ideal rotation requires the knowledge of the gradient at the same point; however an approximate rotation can be performed by adopting, for instance, the values of the gradient evaluated in the previous point of the Markov chain. It is, nonetheless, correct to state, that in most applications only a small subset of parameters is likely to cause most of the response variability, whereas a large part of parameters will have an insignificant effect on the solution, i.e. the gradient has few dominant components.

At each point of the Markov chain, the gradient is not determined directly, as it is the case for finite difference procedures or direct differentiation procedures, but by random sampling in the close neighbourhood of the current chain point, $\boldsymbol{x} = \tilde{\boldsymbol{x}}$. The following function difference, $\mathbf{b}^{(j)}(\tilde{\mathbf{x}}) = f(\tilde{\mathbf{x}} + \gamma \cdot \mathbf{R}^{(j)}) - f(\tilde{\mathbf{x}})$ is then computed from simulation where the parameter $\gamma > 0$ controls the width of scatter points $\mathbf{x}^{(j)}$ around the reference point $\tilde{\boldsymbol{x}}$ and R is a vector full of i.i.d. standard normal random variables. The components of the derivative at the point $\tilde{\boldsymbol{x}}$ are:

$$d_k^{(j)}(\tilde{\boldsymbol{x}}) = \left.\frac{\partial f(\boldsymbol{x})}{\partial x_k}\right|_{\boldsymbol{x}=\tilde{\boldsymbol{x}}} \approx \frac{b_k^{(j)}(\tilde{\boldsymbol{x}})}{\gamma \cdot R_k^{(j)}} \tag{2}$$

The most important gradient components can be estimated from certain statistics as shown in Ref [3]. Finally the following estimators can be used to calculate the functional ν_i as well as the Morris importance measures, μ_i:

$$\nu_i = \int_{H^p} \left(\frac{\partial f(\boldsymbol{x})}{\partial x_i}\right)^2 dx \approx \frac{1}{N_{chain}} \sum_j \left(\frac{b_k^{(j)}(\tilde{\boldsymbol{x}})}{\gamma \cdot R_k^{(j)}}\right)^2; \qquad \mu_i \approx \frac{1}{N_{chain}} \sum_j \left|\frac{b_k^{(j)}(\tilde{\boldsymbol{x}})}{\gamma \cdot R_k^{(j)}}\right| \tag{3}$$

where N_{chain} represents the length of the Markov chain with the points distributed according the multivariate normal distribution.

In conclusion, this approach allows a significant reduction of the computational efforts required to perform the global SA especially for applications involving a high number of variables.

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ON THE DETERMINATION OF THE POWER SPECTRUM OF RANDOMLY EXCITED OSCILLATORS VIA STOCHASTIC AVERAGING: AN ALTERNATIVE PERSPECTIVE

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ABSTRACT

An approximate formula which utilizes the concept of conditional power spectral density (PSD) has often been employed to determine the response PSD of stochastically excited nonlinear systems. Although this expression has been used in numerous applications, its derivation has been so far treated in a rather heuristic, even "unnatural" manner. Indeed, the formula is based on the notion of the conditional PSD, and its mathematical legitimacy has been based on somewhat "arm-waving" arguments. In this paper, a perspective on the legitimacy of this formula is provided by utilizing spectral representations both of the excitation and of the response processes of the nonlinear system. The orthogonality properties of the sinusoidal functions which are involved in the representations are utilized. Furthermore, not only stationarity but ergodicity of the system response is invoked. In this context, the nonlinear response PSD can be construed as a superposition of the PSDs which correspond to equivalent response amplitude dependent linear systems. Next, relying on classical excitation-response PSD relationships for these linear systems leads, readily, to the derivation of the formula for the determination of the PSD of the nonlinear system.

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CHALLENGES AND OPPORTUNITIES FOR STRUCTURAL IDENTIFICATION AND MONITORING USING SMART SENSORS

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1. Background

Structural health monitoring (SHM) provides the means for capturing structural response and assessing structural condition for a variety of purposes. For example, the information from an SHM system can be used to fine-tune idealized structural models, thereby allowing more accurate prediction of the response due to extreme loading conditions, such as earthquakes and typhoons. SHM also can be used to characterize loads in situ, which can allow the detection of unusual loading conditions as well as validate the structure's design. In addition, real-time monitoring systems can measure the response of a structure before, during and after a natural or man-made disaster, and may be used in damage detection algorithms to assess the post-event condition of a structure.

Given the size and complexity of many civil structures, a large network of sensors is usually required to adequately assess the structural condition. Traditional structural monitoring systems have been moving in the direction of dense deployment in recent years; however, the cost of installation can be thousands of dollars per sensor channel, and the amount of data generated by such a system can render the problem intractable. Networks of wireless smart sensors have the potential to improve SHM dramatically by allowing for dense networks of sensors employing distributed computing to be installed on a structure. As defined herein, a smart sensor is a battery-powered sensing node with a micro-processor, memory, and a radio transmitter.

While smart sensor technology has been commercially available for nearly a decade, full-scale implementation has been lacking with the exception of a few short-term demonstration projects [1,2]. This slow progress is due primarily to (i) the lack of an adequate sensing platform and (ii) the fact that programming smart sensors is extremely complex, putting the use of these devices for all but the simplest tasks out of the reach of most engineers. Moreover, critical issues inherent in wireless smart sensor networks (WSSNs), such as synchronized sensing and data loss, must be addressed. In addition, the numerical algorithms required for system identification and damage detection must be implemented on sensor nodes which have limited resources. The result is that SHM applications require complex programming, ranging from network functionality to algorithm implementation. Applications software development is made even more difficult by the fact that many smart sensor platforms employ special-purpose operating systems without support for common programming environments. The extensive expertise required to develop SHM applications has severely limited the use of smart sensing technology.

This paper presents an open-source hardware and software framework for structural health monitoring using WSSN. This framework provides the infrastructure necessary to obtain high-quality response data and to transport it reliably across the sensor network, as well as a broad array of SHM algorithms (see http://shm.cs.uiuc.edu/software.html).

2. Wireless smart sensor platform

The wireless smart sensor platform used in this research is the Imote2 (see Fig. 1), which is uniquely-suited to the demands of SHM applications. It has a low-power X-scale processor (PXA27x) with variable processing speed to optimize power consumption. It incorporates a ChipCon 2420 802.15.4 radio with an onboard antenna (Antenova Mica SMD). The onboard memory of the Imote2 is one of the features that sets it apart from other wireless sensor platforms and allows its use

for the high-frequency sampling required for dynamic structural monitoring. It has 256 KB of integrated RAM, 32 MB of external SDRAM, and 32 MB of flash memory.

A new, versatile sensor board to interface with the Imote2 has been designed that is tailored to the requirements of SHM applications (see Fig. 1). This SHM Accelerometer board (SHM-A) allows 3 axes of acceleration measurement with user programmable anti-aliasing filters. This sensor board has excellent sampling rate accuracy and flexibility. Current versions of the board have incorporated temperature, light and humidity sensors to provide information that is critical in establishing a comprehensive assessment of the structural condition.

3. Service-oriented software framework

With the exponential growth in available computing power over the last 50 years, the complexity of computer software has likewise increased dramatically. Advances in the fields of programming language design and software engineering allow application developers to deal with this complexity by dividing the software system into smaller, manageable parts. Following this approach, enabling services and service-based applications specifically designed for the address the challenges of using smart sensors for structural health monitoring have been developed and made available at the Illinois SHM Project website ([2]; also see http://shm.cs.uiuc.edu).

3. Jindo bridge deployment

To demonstrate the efficacy of the proposed framework, results are presented for a WSSN deployment on the new Jindo Bridge, a cable-stayed bridge in South Korea with a 344m main span (see Fig. 2). This tri-lateral collaboration between University of Illinois, KAIST, and the University of Tokyo constitutes world's largest deployment of wireless smart sensors (70 nodes with 420 sensors) to date for civil infrastructure monitoring. This project signifies a new paradigm for structural health monitoring that is leading to dramatic improvements over existing capabilities.

Figure 1. Twin spans of Jindo Bridge with the newer span on the left.

4. Conclusions

Leveraging this hardware/software framework allows engineers to focus attention on advancement of SHM approaches and the development SHM systems without having to concern themselves with low-level networking, communication, and numerical sub-routines.

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STOCHASTIC SYSTEMS WITH STIFFNESS DEGRADATION DUE TO DAMAGE ACCUMULATION.

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1. Introduction

In the presentation a coupled response-degradation problem for a multidimensional vibrating system is analyzed. The analysis allows to account for the effect of stiffness degradation (during the vibration process) on the response and, in the same time, gives the actual stress values for estimation of damage accumulating in the system.

As is well known, dynamics excitation of engineering systems (including randomly varying excitation) causes variable stress generated in mechanical/structural members and, in the consequence, irreversible changes amplitudes in the material structure. These changes, known as damage accumulation, may have different physical content. But, despite the diversity of underlying physical/mechanical phenomena, it is useful to describe them jointly within a single model relating the rate of damage evolution at time with applied stress. Models of this type operate with a certain damage measure $D(t)$, which characterizes a damage state at time t. It is usually assumed that $D(t)$ is on the interval $[0, D^*]$, where D^* denotes a critical damage, and that is a non-decreasing function of time. In some situations (e.g. in the case of fatigue accumulation) external actions and generated stresses can be conveniently related to discrete values of time (e.g. by N, the number of cycles).

Since the variable stress causing damage (and, in the consequence, stiffness degradation) are generated by a vibratory system it is natural to formulate jointly the system dynamics and damage accumulation. Such an analysis allows to account the effect of stiffness degradation during the vibration process on the response and, in the same time, gives the actual stress values for estimation of damage.

2. General formulation

The coupled response-degradation problem can be formulated in the following form:

$$\ddot{\mathbf{Y}}(t)+\mathbf{F}[\mathbf{Y}(t),\mathbf{D}(t),\mathbf{X}(t,\gamma)]=0 \tag{1}$$

$$\mathbf{Q}[\mathbf{D}(t),\mathbf{Y}(t)]=0 \tag{2}$$

$$\mathbf{Y}(t_0)=\mathbf{Y}_0,\ \dot{\mathbf{Y}}(t_0)=\dot{\mathbf{Y}}_0,\ \mathbf{D}(t_0)=\mathbf{D}_0 \tag{3}$$

where $\mathbf{Y}(t)$ is an unknown response process, $\mathbf{D}(t)$ is a degradation process, $\mathbf{F}[.]$ is the given function of indicated variables satisfying the appropriate conditions for the existence and uniqueness of the solution, $\mathbf{X}(t,\gamma)$ is the given stochastic process characterizing the excitation; $\gamma \in \Gamma$, and Γ is the space of elementary events in the basic scheme (Γ, B, P) of probability theory, $\mathbf{Q}[.]$ symbolizes the relationship between degradation and response process; its specific mathematical form depends on the particular situation; $\mathbf{Y}_0, \dot{\mathbf{Y}}_0, \mathbf{D}_0$ are given initial values of the response and degradation, respectively.

An important special class of the response-degradation problems is obtained if relationship (2) takes the form of differential equation, that is, equations (1), (2) are

$$\ddot{\mathbf{Y}}(t)+\mathbf{F}[\mathbf{Y}(t),\mathbf{D}(t),\mathbf{X}(t,\gamma)]=0 \tag{4}$$

$$\dot{\mathbf{D}}(t)=\mathbf{G}[\mathbf{D}(t),\mathbf{Y}(t)] \tag{5}$$

where **G** is the appropriate function specifying the evolution of degradation; its mathematical form is inferred from the elaboration of empirical data, or - it is derived from the analysis of the physics of the process. In equation (5) dependence on $\mathbf{Y}(t)$ can be in general regarded in more relaxed sense than it is usual. Degradation rate $\dot{\mathbf{D}}(t)$ may depend on the actual values of $\mathbf{Y}(t)$, but it can also depend on some functionals of $\mathbf{Y}(t)$; for example - on the integral of $\mathbf{Y}(\tau), \tau \in [t_0, t]$. In considered here damage degradation problems the damage measure $\mathbf{D}(t)$ depends on the stress range i.e. quantity related to $\mathbf{Y}_{\max} - \mathbf{Y}_{\min}$.

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STOCHASTIC INSTABILITY OF CARBON NANOTUBES VIA NONLOCAL CONTINUUM MECHANICS

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1. Introduction

Dynamic stability of distributed systems has been an object of considerable attention over the past half of century. Numerous papers are available of isotropic and laminated beams, shafts, plates and shells under periodic and random forces. Most of papers have applied finite dimensional or modal approximations in analysis of vibration and stability. The Liapunov direct method is a quite different approach and can be successfully used to analyze continuous systems described by partial differential equations. A significant advantage is offered by the method in that the equations of motion do not have to be solved in order to examine the stability. An application of nonlocal continuum model to representative problems of nanotechnology was demonstrated in [1]. A model based on nonlocal continuum mechanics was applied to solve the buckling of multiwalled nested carbon nanotubes [2]. The detailed study on the flexural wave dispersion in single-walled nanotubes on the basis of beam models in a wide range of wave numbers was presented [3]. It was shown that the vibration analysis results based on nonlocal mechanics are in agreement withthe experimental reports in the field [4]. Based on the Donnel-Vlasov shell theory a double- elastic shell model was presented for the parametric vibrations of double-walled carbon nanotubes under time-dependent membrane forces of thermal origin [5]. The paper is concerned with the stochastic parametric vibrations of micro- and nano-rods based on Eringen's theory and Euler-Bernoulli beam theory

2. Problem formulation

The theory of nonlocal continuum mechanics assumes that the stresses at a given reference point are functions of the strain state of all points in the body. In this way the internal length scale enters into constitutive equations as a material parameter. Adopting Eringen's nonlocal elasticity [6] the nondimensionalized dynamic equation of a short nanotube has the form

$$w_{,tt} + 2\beta w_{,t} + \left(f_o + f(t)\right)w_{,xx} + w_{,xxxx} + \varepsilon\left[w_{,ttxx} + 2\beta w_{,txx} + \left(f_o + f(t)\right)w_{,xxxx}\right] = 0 \tag{1}$$

where ε - the nondimensional small scale parameter, w - the transverse beam displacement, β - viscous damping coefficient, f_o - constant axial force, $f(t)$ - time-dependent component of axial force. The instability problem is solved for simply supported edges. The trivial solution of Eq. (1) is almost sure asymptotically unstable if the measure of disturbed solution tends to infinity with probability 1.

3. Stability analysis and results

In order to examine instability we construct the energy-like Liapunov functional of the form

$$V = \frac{1}{2}\int_0^1 \left[v^2 + 2\beta vw + 2\beta^2 w^2 + \varepsilon\left(v_{,x}^2 + 2\beta v_{,x} w + 2\beta^2 w^2\right) + w_{,xx}^2 - f_o\left(w_{,x}^2 + \varepsilon w_{,xx}^2\right)\right] dx \tag{2}$$

If the classical condition for static buckling is fulfilled the functional (2) is positive-definite and a measure of distance can be chosen as the square root of functional. If trajectories of the forces are physically realizable processes the classical calculus is applied to calculation and we have

(3) $$\frac{dV}{dt} = -2\beta V + 2U$$

where an auxiliary functional is known. In order to find a function λ satisfying inequality

(4) $$U \geq \lambda V$$

In order to find λ we solve Euler auxiliary problem and obtain the first order differential inequality with respect to functional V. The sufficient condition of the almost sure instability is as follows

(5) $$\beta \leq \left\langle \min_{n=1,2,..} \frac{\left|\beta^2 + f(t) n^2\pi^2/2\right|}{\sqrt{\beta^2 + n^2\pi^2\left[n^2\pi^2/\left(1+\varepsilon n^2\pi^2\right) - f_o\right]}} \right\rangle$$

where $\langle\ \rangle$ - mathematical averaging. Based on the formulation obtained instability domains are calculated.

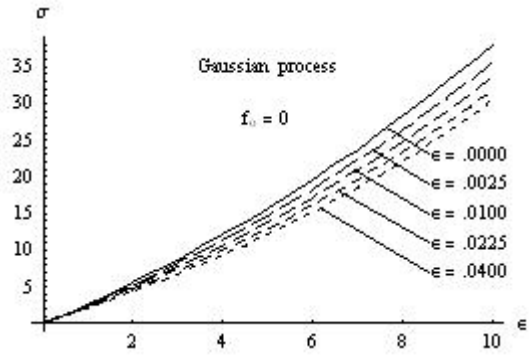

Figure 1. Changes of instability domains with dimensionless scale parameter ε.

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ON MY ADVENTURES WITH STOCHASTIC MECHANICS

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When we pass 70 years of our life, even though we do not think that it is something extraordinary, some retrospective thoughts naturally come to our mind. They are stimulated by a simple observation that things around have changed tremendously in comparison with our experiences fifty or even twenty years ago. As the professional aspect is concerned, one has to realize that the formal employment is, in fact, over. So, one has to accustom to another role in the society – symbolized by a new "title": emeritus. At this new stage of life it is not very likely that someone will encourage you to new challenges since the youngsters are believed to have greater capability. One should look for other, and perhaps – easier things to do. No doubt that reflections on the past are easier.

In the story below I wish to tell a reader – how it happened that I joined my life with stochastics, and more specifically – with stochastic mechanics. I also wish to mention some "events" on my professional road which made my life interesting.

When in 1960 I graduated from the Department of Mathematics & Physics of the Warsaw University (with the masters degree in mathematics) I got a job in the research institute for energy supply (Instytut Energetyki) in Warsaw. However, after few months I felt that this place did not give me sufficient satisfaction, so I kept my eyes open to find another place for my professional development. Once in the spring 1961 one of my colleagues from the University phoned me and said that the Institute of Fundamental Technological Research of the Polish Academy of Sciences (particularly: Prof. S. Kaliski working in dynamic elasticity theory and related fields) was interested in fresh graduates (with master's degree) for doctoral studies. This message delighted me. After few months (since July 1, 1961) I had become a graduate for Ph.D. in the Institute named above (in which I have been working up to now).

In the first few months within my doctoral study my duty was to study mechanics, some branches of physics, theory of wave propagation and some other topics. One day (probably about the spring 1962) Professor S. Kaliski in short conversation with me (he was a busy and rather a formal man) said something like that: "it is a proper time you started to work on a specific problem which may lead you to Ph.D. degree; in this Institute nobody works in stochastic applications, but this is an important and promising topic". And then he continued "the problem I have for you is the following: imagine that you have an elastic wave incident at randomly rough surface which separates two different elastic half-spaces. Since the surface is randomly rough the wave will scatter and the scattered wave field will be random. So your task is to solve the problem of scattering of elastic wave at random surface. Later you may also think on the scattering of the Rayleigh surface waves at randomly rough boundary of an elastic half-space".

As I can recollect today, it was probably the most stressful conversation in my life and … a perspective for a very hard time through all coming weeks and months (!). Even the simplest questions which I asked myself in the first weeks and months (in 1962) after this "conversation" were terrible. For example: what does it mean "random surface" ? How can one understand randomness of a scattered wave field? In the language of mathematics I have a random/stochastic boundary value problem for the elasticity theory equations – how can one formulate properly such a problem? In addition, there was nobody around with whom I could discuss my questions and … share my frustrations. However, there is no doubt that the problem posed by Prof. Kaliski was not only challenging but also intriguing, attractive and important for applications.

This was the beginning of my way to stochastic mechanics!

A reader of this story can easily imagine that my work on the doctoral problem required much effort. It was also a time of continuous studying the analogous problems occurring in radiophysics, geophysics, acoustics. Searching for analogies has always been a power of science. In the middle of 1965 my doctoral thesis was nearly completed and its public defence took place in May 1966. This was a great relief; especially that at the same time (August 1965) I met Anna who one year later became my wife. So, my life entered a new, exciting and happy stage lasting until today.

The work on doctoral thesis introduced me into a wider circle of problems, namely – stochastic wave propagation which has become my field of research during quite a long time (until early 1980-ties). I extended my interest to the problems of wave propagation in random media. Such problems are mathematically modelled by the partial differential equations with random coefficients, or – treated in a different way when the wave transmitting medium is a composition of a matrix medium and randomly distributed inclusions (this topic was a subject of my habilitation in 1974; main results were published in Acta Mechanica, 1976). An important "event" in my life associated with stochastic waves was my research fellowship to the USA in the academic year 1970/71, especially, my stay in the Courant Institute of Mathematical Sciences of New York University. It was really exciting and challenging to live in New York city and be a fellow of this excellent mathematical institute with many famous names including Courant himself, K. Friedrichs, P. Lax and J.B. Keller who in sixties published the best papers on wave propagation in random media.

About the end of seventies/beginning of eighties (of XX-th century) I had noticed that the stochastic wave propagation is not the only field of my scientific interest. Also, I felt that perhaps, I should turn my research more to the main stream of research in the Institute (which was mechanics of materials and structures). It happened that I already was somehow prepared to such a change. The academic year 1975/76 I spent, as a Talbot–Crosbie research fellow, in the Department of Mechanical Engineering of Glasgow University working with Professor J.D. Robson and Dr D.B. Macvean on random vibration of road vehicles travelling with varying velocity. The topic of random vibration fitted to my interest in stochastic differential equations, whereas degrading effects of dynamics, such as fatigue were important for the reliability of engineering materials and structures.

I think I should add here that at the same time I was afraid of "loosing" my quite an extensive knowledge and research experience in the analysis of wave propagation. So, around 1980 I decided to write a book which would summarize my and other existing results on stochastic waves. Such a book was published in Polish in 1982 ("Fale Stochastyczne", PWN, Warsaw) and its extended version – in English: "Stochastic Wave Propagation", Elsevier, 1985. Undoubtedly, this was a good decision; work on this book gave me much intellectual satisfaction and to some extent made my name recognizable.

So, starting from early eighties my research interest has almost entirely been concentrated on my new field: stochastic dynamics of engineering systems and random fatigue of materials. This field having been quite new and attractive about thirty years ago, still remains a lively and important subject of scientific endeavour. My long work in this domain has brought me much joy and intellectual satisfaction; it also allowed me to participate in the international research collaboration.

It is not my intention to write here about all my exciting connections with various research institutions and many professional colleagues. However, I feel I should make some exceptions.

First, I wish to mention my visiting professorship at the Technical University of Denmark (Lyngby) in 1985 due to the invitation of Professor Ove Ditlevsen; it was great to do joint research with Ove, to lecture on stochastic differential equations for applications and to write the Lecture Notes on this topic (nicely published at Lyngby), which later – modified and extended – were published as the book "Stochastic Differential Equations with Applications to Physics and Engineering", Kluwer, Dordrecht, 1991.

The second important "event" is my long-standing, very effective collaboration with Professor Billie F. Spencer at the University of Notre Dame (Indiana – USA). This collaboration started in 1990 when I was offered there the Melchor Endowed Visiting Professorship for 1990/91. Later I held – the Massman Visiting Professorship for the academic year 1997/98. In the meantime (1993-97) we did research with Bill on random fatigue within the joint USA – Poland research project of Maria Skłodowska-Curie. This decade of joint work resulted in several research papers and two books: Sobczyk K., Spencer B.F., "Random Fatigue: from Data to Theory", Academic Press, Boston, 1992; Sobczyk K., Kirkner D.B., "Stochastic Modelling of Microstructures", Birkhauser, Boston, 2002. This long time of collaboration was really a great experience for me both, professionally and socially.

And, finally, writing about my ties with the USA science I wish to mention my very close relations with Professor Y.K. Lin – a renowned scientist in stochastic dynamics leading, during above than twenty years, the Stochastic Research Center at the Florida Atlantic University (Boca Raton). Those who have met Mike (a popular first name of Y.K. Lin) are impressed by his great culture and honesty in the scientific work. My two visits to Boca Raton (in 1988 and 2004) as the invited visiting professor were scientifically highly valuable and enjoyable.

The second "channel" of my ties with the international research activity was the scientific conferences. Presentation of my research work at various scientific meetings has given me a strong feeling of the unity of science – in spite of various human experiences. The conferences which I regard as especially rewarding for me were the following: the Workshop at Lyngby (1982) organized by Professor Ove Ditlevsen; the Weibull Memoriam – IUTAM Symposium in Stockholm (1984) organized by Professors S. Eggwertz and N.C. Lind; the IUTAM Symposia on Stochastic Dynamics: in Igls/Insbruck (1987) organized by Professors G. Schuëller and F. Ziegler; in Torino (1991) organized by Professors N. Bellomo and F. Casciati, and in Trondheim (1995) organized by Professors A. Naess and S. Krenk; the "Spanos conferences" on Stochastic Computational Mechanics organized by Professor P. Spanos (Athens – 1994, Santorini – 1998, Corfu – 2002); the conferences in the USA, e.g. in Blacksburg – Virginia (1988) organized by Professors R.A. Heller and M.P. Singh, in Denver – Colorado (1992) organized by Professors R.B. Corotis and Y.K. Lin, in Worcester – Massachusetts (1996) organized by Professor M.N. Noori, in Notre Dame – Indiana (1998) organized by Professors B.F. Spencer and E.A. Johnson; and finally: the conference on Nonlinear Mechanics and Stochastic Dynamics in Waterloo – Canada (1993) organized by professors: W. Kliemann and S.Namachchivaya.

Stochastic dynamics of engineering systems was also a subject which I had a honour to present in the highly prestigious invited plenary lecture at the XXI-st World IUTAM Congress of Theoretical and Applied Mechanics, Warsaw, 2004 (organized by professors: W. Gutkowski and T. Kowalewski).

However, this what has been especially wonderful during all this long time was my deep feeling that the world community on Stochastic Mechanics constitutes a unique group of friends. Today I greatly appreciate that many of these friends are here in Warsaw – participating in our special meeting – the International Conference on Stochastic Methods in Mechanics: Status and Challenges (Warsaw, September 28-30, 2009).

O MOICH PRZYGODACH Z MECHANIKĄ STOCHASTYCZNĄ

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Kiedy człowiek przekracza 70 lat swojego życia to nawet gdyby nie uważał tego faktu za coś nadzwyczajnego, pewne myśli o charakterze retrospektywnym w sposób naturalny przychodzą mu do głowy. Są one stymulowane przez prostą obserwację, iż świat wokół nas zmienił się ogromnie w porównaniu z naszymi doświadczeniami pięćdziesiąt czy nawet dwadzieścia lat temu. W wymiarze profesjonalnym kończy się czas naszego formalnego zatrudnienia i musimy się przyzwyczaić do nowej roli w społeczeństwie – symbolizowanej przez nowy „tytuł": emeritus. Na tym etapie życia jest mało prawdopodobne, że ktoś będzie zachęcał nas do podjęcia nowych wyzwań, gdyż młodsi – niejako z natury rzeczy – powinni mieć większe możliwości. Należy więc „rozglądać się" za innymi i raczej łatwiejszymi zajęciami. Bez żadnych wątpliwości, refleksje o przeszłości są łatwiejsze.

W tym krótkim eseju pragnę podzielić się z Czytelnikiem o tym jak to się stało, że związałem moje życie ze stochastyką, a dokładniej – z mechaniką stochastyczną. Chciałbym też wspomnieć o pewnych „wydarzeniach" związanych z moją pracą naukową, które przyniosły mi szczególną satysfakcję.

Kiedy w 1960 roku ukończyłem Wydział Matematyczno-Fizyczny na Uniwersytecie Warszawskim (otrzymując stopień magistra matematyki) zacząłem pracować w Instytucie Energetyki w Warszawie. Jednakże, już po kilku miesiącach pracy wyczuwałem, iż nie jest to miejsce, które może dostarczać mi wystarczającą satysfakcję. Byłem więc zainteresowany znalezieniem innego, właściwego miejsca dla mojego rozwoju zawodowego. I wtedy, pewnego dnia na wiosnę roku 1961 zadzwonił do mnie kolega ze studiów mówiąc, że Instytut Podstawowych Problemów Techniki Polskiej Akademii Nauk (a dokładniej: profesor S. Kaliski – pracujący wtedy w zakresie dynamicznej teorii sprężystości i w dziedzinach pokrewnych) jest zainteresowany przyjęciem „świeżych" magistrów na studia doktoranckie. Ta wiadomość bardzo mnie zainteresowała. Po kilku miesiącach (od 1-go lipca 1961 r.) zostałem doktorantem w tym znanym instytucie badawczym (z którym pozostałem związany do dnia dzisiejszego).

W pierwszych kilku miesiącach studiów doktoranckich moje główne zadanie polegało na studiowaniu wybranych działów fizyki, mechaniki, a także matematyki, które miały być potrzebne dla przyszłej pracy badawczej. Pewnego dnia (prawdopodobnie – wiosną 1962 r.) profesor Kaliski w krótkiej rozmowie ze mną (był On człowiekiem zajętym i dość formalnym) powiedział mniej-więcej tak: „przyszedł chyba właściwy czas, aby Pan zaczął pracować nad jakimś konkretnym problemem, co prowadziłoby do doktoratu; w tym instytucie nikt nie pracuje w zastosowaniach stochastycznych (tj. w modelowaniu i analizie rzeczywistych zjawisk losowych), ale jest to kierunek bardzo ważny i „przyszłościowy". I dalej kontynuował: „ mam dla Pana następujący problem: niech sobie Pan wyobrazi, że fala sprężysta pada na powierzchnię z losowymi nierównościami (chropowatościami), która to powierzchnia rozdziela dwie różne półprzestrzenie sprężyste. Ponieważ powierzchnia jest chropowata, fala będzie ulegać rozproszeniu, ale falowe pole rozproszone będzie losowe (bo nierówności są rozłożone losowo). Tak więc, Pana zadanie polega

na rozwiązaniu problemu rozpraszania fali sprężystej na powierzchni geometrycznie losowej. Później powinien Pan pomyśleć o rozpraszaniu powierzchniowej fali Rayleigh'a na powierzchni losowej będącej granicą półprzestrzeni sprężystej".

Na tyle ile dziś mogę sobie przypomnieć, rozmowa ta była najprawdopodobniej – jedną z najbardziej stresujących rozmów w moim życiu i ... perspektywą ciężkiej pracy w nadchodzącym czasie (!). Nawet najprostsze pytania jakie stawiałem sobie po tej „rozmowie" w pierwszych tygodniach i miesiącach 1962 roku były straszne; na przykład: co oznacza „powierzchnia losowa"? jak należy rozumieć losowość rozproszonego pola falowego? Postawiony problem oznacza, iż – matematycznie rzecz ujmując – mam losowy/stochastyczny problem brzegowy dla równań teorii sprężystości; jak sformułować poprawnie taki problem? Dodatkowo, nie było wokół mnie nikogo, z kim mógłbym przedyskutować moje pytania, a także ... dzielić się moimi frustracjami. Tym niemniej, bez wątpienia problem postawiony przez profesora Kaliskiego był nie tylko dla mnie wyzwaniem, ale był także atrakcyjny naukowo i ważny w zastosowaniach. Był to początek mojej drogi do mechaniki stochastycznej!

Czytelnik tego eseju może łatwo wyobrazić sobie, że moja praca nad problemem doktorskim wymagała wiele wysiłku. Był to także dla mnie czas wytrwałego studiowania różnych problemów dotyczących propagacji i rozpraszania fal w radiofizyce, geofizyce i akustyce. Poszukiwanie analogii było zawsze siłą nauki. W połowie roku 1965 moja praca doktorska była na ukończeniu, a jej publiczna obrona odbyła się w maju 1966 roku. Przyszedł czas przyjemnego odprężenia i satysfakcji; tym bardziej, że mniej więcej w tym samym czasie (sierpień 1965) spotkałem osobę, która rok później została moją żoną. Tak więc życie moje weszło w nowy, interesujący i szczęśliwy okres trwający do dnia dzisiejszego.

Praca badawcza nad doktoratem wprowadziła mnie w szerszy krąg problemów badawczych, mianowicie w zagadnienia propagacji fal stochastycznych, które pozostawały głównym przedmiotem mojego zainteresowania do wczesnych lat osiemdziesiątych. Badania te dotyczyły propagacji fal w ośrodkach stochastycznych. Zjawiska falowe w takich ośrodkach są matematycznie opisywane przez równania różniczkowe cząstkowe z losowymi współczynnikami; mogą też być charakteryzowane w inny sposób, jeżeli ośrodek przenoszący ruch falowy jest mieszaniną/kompozycją ośrodka macierzystego i losowo rozłożonych inkluzji (te problemy były przedmiotem mojej habilitacji w 1974 roku; główne rezultaty zostały opublikowane w Acta Mechanica, 1976 r.). Ważnym „wydarzeniem" w moim życiu związanym z tym nurtem badań było moje stypendium badawcze w USA w roku akademickim 1970/71, a nade wszystko mój pobyt w Instytucie Matematycznym im. Couranta Uniwersytetu Nowojorskiego. Było czymś bardzo interesującym i ekscytującym mieszkać w Nowym Jorku i pracować w tym doskonałym instytucie matematycznym w otoczeniu wielu sławnych uczonych, takich jak m.in.: sam R. Courant, M.D. Donsker, K. Friedricks, P. Lax i J.B. Keller – który w latach sześćdziesiątych publikował najlepsze prace na temat propagacji fal w ośrodkach stochastycznych.

Na przełomie lat siedemdziesiątych i osiemdziesiątych (XX wieku) coraz częściej odczuwałem, że propagacja fal stochastycznych nie jest jedynym polem moich zainteresowań naukowych. Myślałem też, że byłoby pożyteczne zbliżenie moich badań do głównego nurtu badań w Instytucie (którym była mechanika materiałów i konstrukcji). Tak się złożyło, że w jakimś stopniu byłem już przygotowany do takiej zmiany. Rok akademicki 1975/76 spędziłem bowiem jako Talbot-Crosbie Research Fellow na Wydziale Mechanicznym Uniwersytetu w Glasgow pracując z Prof. J.D. Robsonem i Dr. D.B. Macvean'em nad zagadnieniem drgań losowych pojazdów drogowych poruszających się ze zmienną prędkością. Dziedzina drgań losowych/stochastycznych odpowiadała dobrze moim zainteresowaniom stochastycznymi równaniami różniczkowymi, podczas gdy różne efekty degradacji elementów konstrukcji na skutek drgań, np. zniszczenie zmęczeniowe, były ważne dla różnych zastosowań.

Myślę, że powinienem w tym miejscu dodać, iż jednocześnie z chęcią zmiany moich zainteresowań obawiałem się „utraty" mojej wiedzy i doświadczenia badawczego w zakresie propagacji fal. Około 1980 r. postanowiłem więc napisać książkę, która sumowałaby moje i inne rezultaty badawcze dotyczące fal stochastycznych. Książka taka została opublikowana w języku polskim w 1982 r. („Fale stochastyczne, PWN, Warszawa), zaś jej rozszerzona wersja anglojęzyczna „Stochastic Wave Propagation" została wydana przez Elsevier w 1985 r. Praca nad tą książką dostarczyła mi dużo intelektualnej satysfakcji i w jakimś stopniu, uczyniła moje nazwisko rozpoznawalnym.

Tak więc począwszy od wczesnych lat osiemdziesiątych moje zainteresowania badawcze koncentrowały się niemal całkowicie na nowej dziedzinie: dynamice stochastycznej układów technicznych i stochastycznej analizie zniszczenia zmęczeniowego materiałów. Ta dziedzina – będąca nową i atrakcyjną w owym czasie pozostaje ciągle żywym i ważnym obiektem dociekań naukowych ze względu na jej doniosłe znaczenie aplikacyjne. Moja praca w tej problematyce w ostatnim długim okresie czasu przyniosła mi przyjemność i dużą satysfakcję; pozwalała mi też w pełni uczestniczyć w międzynarodowej współpracy badawczej.

Nie jest moją intencją opisywanie tutaj wszystkich moich wielce ekscytujących powiązań naukowych z różnymi instytucjami badawczymi i z wieloma zaprzyjaźnionymi uczonymi z różnych krajów. Wyczuwam jednakże, że powinienem zrobić pewne wyjątki.

Chcę wspomnieć mój 3-miesięczny pobyt w Danii w 1985 roku – w charakterze profesora wizytującego w Duńskim Uniwersytecie Technicznym (Lyngby k/Kopenhagi) na zaproszenie prof. Ove Ditlevsen'a, wybitnego specjalisty w zakresie niezawodności konstrukcji i mechaniki stochastycznej. Z przyjemnością wspominam wspólną pracę badawczą z Ove, wykłady na temat stochastycznych równań różniczkowych dla zastosowań oraz jednoczesne pisanie „skryptu" (ang. Lecture notes), który został szybko i pięknie wydany przez wymieniony wyżej Uniwersytet. Jego zmodyfikowana i rozszerzona wersja przyjęła później formę książki: Sobczyk K., „Stochastic Differential Equations with Applications to Physics and Engineering", Kluwer Acad. Publishers, Dordrecht, 1991.

Drugie ważne „wydarzenie" w mojej współpracy międzynarodowej to moja wieloletnia i bardzo efektywna współpraca z Prof. Billie F. Spencer'em – Uniwersytet Notre Dame (Indiana, USA). Ta naukowa przygoda rozpoczęła się w 1990 roku kiedy ten Uniwersytet zaoferował mi specjalne stanowisko profesora wizytującego (the Endowed Melchor Visiting Professorship) na rok 1990/91. Po kilku latach podobne stanowisko (the Endowed Massman Professor in Engineering) otrzymałem na rok 1987/88. W międzyczasie (1993-97) razem z Bill'em realizowaliśmy wspólne badania (dotyczące stochastycznej analizy zniszczenia zmęczeniowego) w ramach wspólnego (USA – Polska) projektu badawczego im. Marii Skłodowskiej-Curie. Ta dekada współpracy zaowocowała kilkoma pracami w periodykach międzynarodowych i dwiema książkami: Sobczyk K., Spencer B.F., „Random Fatigue: from Data to Theory", Academic Press, Boston 1992; Sobczyk K., Kirkner D.B., „Stochastic Modelling of Microstructures", Birkhauser, Boston, 2002. Współpraca ta była dla mnie autentycznie wielkim doświadczeniem zarówno w sferze organizacji i prowadzenia badań jak i w sferze szeroko rozumianej kultury i relacji międzyludzkich. I wreszcie, gdy mówię o związkach z nauką amerykańską, powinienem wspomnieć moje bliskie relacje z profesorem Y.K. Lin'em – wybitnym specjalistą w dynamice stochastycznej, prowadzącym od połowy lat osiemdziesiątych Centrum Badań Stochastycznych (ang. Stochastic Research Center) w Uniwersytecie o pełniej amerykańskiej nazwie: Florida Atlantic University (Boca Raton). Ci, którzy znają prof. Lin'a, wysoko cenią Jego wielką kulturę i uczciwość w pracy badawczej. Moje pobyty w Boca Raton jako profesor wizytujący (w 1988 r. oraz w 2004 r.) i dyskusje z prof. Lin'em wspominam dzisiaj z prawdziwą przyjemnością.

Drugim nurtem moich związków z badaniami prowadzonymi w różnych ośrodkach na świecie były konferencje naukowe. Wygłaszane przez mnie referaty (a często także udział w komitetach naukowych) dawało mi poczucie jedności nauki – niezależnie od różnic w ludzkich doświadczeniach. Wymienię tutaj konferencje, uczestnictwo w których miało dla mojego życia naukowego istotne znaczenie. Oto one: spotkanie (workshop) w Lyngby (1982) organizowane przez prof. Ove Ditlevsen'a, Weibull Memoriam – IUTAM Symposium w Sztokholmie (1984) organizowane przez profesorów: S. Eggwertz'a i N.C. Lind'a, sympozja IUTAM-owskie nt. Dynamiki Stochastycznej: w Igls/Insbruck (1987) organizowane przez profesorów G. Schueller'a i F. Ziegler'a, w Turynie (1991) organizowane przez profesorów N. Bellomo i F. Casciati'ego oraz w Trondheim (1995) organizowane przez profesorów A. Naess'a i S. Krenk'a; konferencje poświęcone „obliczeniowej" mechanice stochastycznej organizowanej w Grecji przez profesora P. Spanos'a (z Rice University w Teksasie) – w Atenach (1994), Santorini (1998), Corfu (2002); konferencje w USA: w Blackburg – Virginia (1988) organizowana przez profesorów R.A. Heller'a i M.P. Singh'a, w Denver – Colorado (1992) organizowane przez profesorów R.B. Corotis'a i Y.K. Lin'a, w Worcester – Massachusetts (1996) organizowane przez profesora M.N. Noori'ego, w Notre Dame – Indiana (1998) organizowane przez profesorów B.F. Spencer'a i E.A. Johnson'a i wreszcie – konferencja nt. Mechaniki nieliniowej i dynamiki stochastyzcnej w Waterloo – Kanada (1993) organizowana przez profesorów: W. Kliemann'a i S. Namachchivaya.

Dziedzina mojej pracy badawczej – Dynamika stochastyczna układów technicznych była też przedmiotem, który miałem zaszczyt prezentować w moim wysoko prestiżowym referacie plenarnym (na zaproszenie Komitetu Organizacyjnego) na XXI-ym Światowym Kongresie IUTAM (Intern. Union of Theoretical and Applied Mechanics), Warszawa, 2004 (organizatorzy – profesorowie: W. Gutkowski I T. Kowalewski).

Jednakże, tym co było wyjątkowo wspaniałe w tym długim czasie mojej pracy badawczej i międzynarodowej współpracy było moje silne odczucie, że międzynarodowa społeczność badaczy w mechanice stochastycznej tworzy unikatową grupę przyjaciół. Dzisiaj jest mi bardzo miło, że wielu z tych przyjaciół jest tutaj w Warszawie – uczestnicząc w naszej specjalnej konferencji międzynarodowej nt. „Stochastic Methods in Mechanics: Status and Challenges" – Warszawa, 28 – 30 września 2009 r.

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